Lecture 1: Outline

- Administrative stuff.
- Some History.
- LP flavors.
History of Optimization

Fermat, 1638; Newton, 1670

\[
\min f(x) \quad x: \text{ scalar}
\]
\[
\frac{df(x)}{dx} = 0
\]

Euler, 1755

\[
\min f(x_1, \ldots, x_n)
\]
\[
\nabla f(x) = 0
\]

Lagrange, 1797

\[
\min f(x_1, \ldots, x_n)
\]
\[
s.t. \ g_k(x_1, \ldots, x_n) = 0, \quad k = 1, \ldots, m.
\]

Euler, Lagrange  Problems in infinite dimensions \((n \to \infty)\), calculus of variations.

Linear Programming (LP)

\[
\text{minimize} \quad 3x_1 + x_2
\]
\[
s.t. \quad x_1 + 2x_2 \geq 2
\]
\[
2x_1 + x_2 \geq 2
\]
\[
x_1 \geq 0
\]
\[
x_2 \geq 0
\]

in vector notation

\[
c = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}
\]

\[
\text{minimize} \quad c'x
\]
\[
s.t. \quad Ax \geq b
\]
\[
x \geq 0
\]
**LP History**

- **George Dantzig, 1947**  Simplex method.
- **Fourier, 1826**  Method for solving system of linear inequalities.
- **de la Vallée Poussin**  Simplex-like method for objective function with absolute values.
- **Kantorovich, Koopmans, 1930s**  Formulations and solution method.
- **von Neumann, 1928**  Game theory, duality.
- **Farkas, Minkowski, Carathéodory, 1870-1930**  Foundations.
  - 1950s  Applications.
  - 1960s  Large Scale Optimization.
  - 1970s  Complexity theory.
- **Khachyan, 1979**  The ellipsoid algorithm.
- **Karmakar, 1984**  Interior point algorithms.

**Applications of LP**

- Transportation (WW II, air traffic control, crew scheduling, etc.)
- Telecommunications (routing, scheduling, resource allocation)
- Manufacturing (production planning, scheduling, resource allocation)
- Medicine, Computational Biology (metabolic networks, protein side-chain packing).
- Engineering.
- Typesetting (\TeX, \LaTeX)
Possible solution outcomes

- There exists a **unique optimal solution**.
- There exist **multiple optimal solutions** (their set being either bounded or unbounded).
- Optimal cost is \(-\infty\) and no feasible solution is optimal (**unbounded problem**).
- Feasible set is empty (**infeasible problem**).

Various LP Flavors

**(General LP)**

\[
\begin{align*}
\text{minimize} & \quad c'x \\
\text{s.t.} & \quad Ax \geq a \\
& \quad Bx \leq b \\
& \quad Dx = d \\
& \quad x_i \geq 0, \quad i \in I \\
& \quad x_j \leq 0, \quad j \in J.
\end{align*}
\]

reduces to

\[
\begin{align*}
\text{minimize} & \quad c'x \\
\text{s.t.} & \quad Ax \geq b
\end{align*}
\]

**(Standard form LP)**

\[
\begin{align*}
\text{minimize} & \quad c'x \\
\text{s.t.} & \quad Ax = b \\
& \quad x \geq 0
\end{align*}
\]

Remark

Every LP can be written in standard form.