Dynamic Loss Aversion, Growth, and Development*

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Abstract

We build a prospect-theoretic model to explain several stylized facts in the development literature. Agents get reference-dependent utility from the income generated by their assets, and are more affected by losses than by gains. Such agents may under-invest in novel or risky assets, leading to unexploited opportunities for high marginal returns, while simultaneously maintaining high holdings of low-return assets that they have owned in the past. There is a range of possible steady-state asset allocations, depending on past ownership, in contrast to conventional models of poverty traps. The provision of insurance against catastrophic loss will have a larger effect in motivating such agents to invest than it would on agents with classical preferences. We show how credit contract design can partially mitigate under-investment while simultaneously encouraging repayment of loans.

(JEL codes: D03, D91, O12, O16)

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1 Introduction

In this article, we propose that loss aversion can explain a number of puzzles and recent findings in development economics. An agent with loss-averse preferences feels losses more intensely than gains. Loss aversion makes agents more averse to certain risks, and it can make agents reluctant to part with what they have in order to invest in something new, leading to stickiness of asset allocations.

In particular, we argue that loss aversion may help explain the prevalence of unexploited high return investment opportunities among farmers and small scale entrepreneurs in the developing world. Consistent with the view that loss aversion inhibits investment, Kremer, Lee, Robinson and Rostapshova (2011) find that shopkeepers who are averse to small gambles with positive expected returns maintain lower inventories. We use our model to show that an agent with loss-averse preferences around investments and consumption has an range of asset levels consistent with steady-state, in contrast to standard models of poverty traps which typically feature a small number of absorbing steady-states.

We argue that loss aversion helps explain recent findings that provision of insurance to farmers can lead them to substantially increase their investment (Mobarak and Rosenzweig, 2012; Karlan, Osei, Osei-Akoto and Udry, 2014).

Loss aversion can also provide one explanation for a findings by Jack et al. (2014): that when farmers who are offered loans for a productive asset are given the opportunity to use that asset as collateral for the loan, they greatly increase take up of loans and repay these loans at thigh rates; that many repay early; and that existing assets are a weak prediction of taking up loans collateralized by existing assets, and that those ownership of an existing, less durable focus of the asset was a key predictor of take up.

In a classical benchmark model (Section 2), agents invest to equalize the risk-adjusted marginal return across assets with each other and with their discount rate. However, we argue that a series of stylized facts (Section 3) about investment, insurance, and migration are difficult to reconcile with the benchmark. After giving some background on loss aversion (Section 4), we of develop a model of investment by loss-averse agents (Section 5).

We then apply a loss aversion perspective to the stylized facts (Section 6), focusing special attention (Section 6.4) on how loss aversion can motivate credit interventions to encourage investment and high repayment rates, connecting with ongoing experimental research by Jack et al. (2014). We conclude (Section 7) with discussions of how the industrial structure of developing countries may exacerbate the effect of behavioral tendencies such as loss aversion, and of the welfare implications of our findings.
2 Classical Model of Investments

We first lay out a classical Ramsey model as a benchmark for investment behavior. Consider an agent who must divide her period-\(t\) wealth \(W_t\) between consumption \(c_t\) and investments \(k_{it}\) in assets indexed by \(i\) that have gross returns \(f_i(k_{it})\) with \(f_i'(\cdot) \geq 0\). This gross return incorporates both the value of the principal, net output, and depreciation.\(^1\) Returns may be stochastic, in which case they are written as \(f_i(k_{it}; \theta_{it}) \equiv f_{it}(k_{it})\), with productivity shock \(\theta_{it}\) for asset \(i\) independently and identically distributed across periods. The uncertainty could include shocks to productivity, such as rainfall, as well as to price, such as market demand shifters. The vector \(\theta_t\) of shocks to period-\(t\) investments is resolved at the beginning of period \(t + 1\).

The agent also has exogenous and predictable labor income \(y_t\). In our specification, she cannot borrow (though borrowing could be incorporated without much impact on the model’s implications). She gets utility from consumption and discounts exponentially with discount factor \(\beta\), so in each period \(t\), her optimization problem is

\[
V(W_t) = \max_{c_t, \{k_{it}\}} u(c_t) + \beta E_t \left[ V(W_{t+1}) \right]
\]

s.t. \(c_t + \sum_i k_{it} \leq W_t\)

\[W_{t+1} := y_{t+1} + \sum_i f_i(k_{it}; \theta_{it})\]

The Euler equation that describes her optimal investment and consumption strategy is, for each asset \(i\):

\[
E_t \left[ \frac{\beta u'(c_{t+1})}{u'(c_t)} f_i'(k_{it}) \right] = 1 \quad \text{if } k_{it} > 0
\]

\[
E_t \left[ \frac{\beta u'(c_{t+1})}{u'(c_t)} f_i'(k_{it}) \right] \leq 1 \quad \text{if } k_{it} = 0.
\]

A commonly used specification is power utility \(u(c) = \frac{c^{1-\sigma}}{1-\sigma}\) with \(\sigma > 0\) (and \(u(c) = \log c\) for \(\sigma = 1\)), which has constant intertemporal elasticity of substitution \(\sigma\). If the agent invests in a risk-free asset with gross return \(f' = R\), then with power utility the Euler equation dictates consumption growth according to

\(^1\)For a financial asset it would include the change in price and any dividends or coupons. For a long-lived physical asset like a machine or livestock, it would include the value net of depreciation, any input costs, and any outputs produced. For an expendable asset such as fertilizer, it would include any change in the value of crops attributable to it.
where \( r \) is the rate of return and \( \delta \) is the discount rate.

For risky assets, basic asset pricing provides a useful interpretation of the Euler equation.\(^2\) The stochastic discount factor
\[ m_{t+1} := \beta u'(c_{t+1})/u'(c_t) \]
gives the marginal rate of substitution at \( t \) between consumption at \( t \) and \( t+1 \); with power utility, it is
\[ m_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^\sigma. \]
The intertemporal elasticity \( \sigma \) serves as coefficient of relative risk aversion
\[ \frac{u''(c)}{cu'(c)}, \]
since agents sum utility linearly across states as well as times.

With risk-free rate \( R \), equation (1) says that the consumption path must satisfy
\[ E_t [m_{t+1}] = \frac{1}{R} \]
even if savings is interior (or equivalently if borrowing is, at rate \( R \)),\(^3\) and investment in asset \( i \), if positive, satisfies

\[
E_t [f_{it}^o(k_{it})] - R = -\frac{\text{Cov}_t [u'(c_{t+1}), f_{it}^o(k_{it})]}{E_t [u'(c_{t+1})]} \]
\[ = \left( \frac{\text{Cov}_t [f_{it}^o(k_{it}), m_{t+1}]}{\text{Var}_t [m_{t+1}]} \right) \left( -\frac{\text{Var}_t [m_{t+1}]}{E_t [m_{t+1}]} \right). \]
\[ \approx -\sigma \cdot \text{StdDev}_t \left[ \ln \frac{c_{t+1}}{c_t} \right] \text{StdDev}_t \left[ f_{it}^o(k_{it}) \right] \text{Corr}_t \left[ m_{t+1}, f_{it}^o(k_{it}) \right]. \]

According to equation (2), assets whose marginal return negatively covary with consumption receive more investment (with lower marginal returns) because they serve as insurance, while those that positively covary make consumption more volatile and so receive less investment. This risk premium of an asset is decomposed in (3) into one term for how returns covary with consumption (the beta of the asset) and one for the volatility of consumption. With power utility and lognormal consumption growth, the risk premium is approximated in (4), so it is greater in magnitude for agents with higher intertemporal elasticity \( \sigma \) (Cochrane, 2005).

Thus, according to the Euler equation, agents invest so as to equalize the risk-adjusted marginal return in all assets, with more investment and lower expected marginal returns for those assets that have high marginal returns in states of the world with low income and consumption.

\(^2\)See, e.g. Cochrane (2005) for an introduction to the Euler equation in finance.

\(^3\)The risk-free rate is well-defined as \( \frac{1}{E_t [m_{t+1}]} \) even if the agent does not have access to a risk-free asset with this rate.
To sharpen the model’s predictions, we calibrate its parameters. We use an annual discount factor of $\beta = .96$. Values around .96 are commonly used in macroeconomics (see, e.g., Golosov and Lucas, 2007), and studies estimating it from life-cycle consumption bound it above .9, even for low-income groups for which estimated $\beta$ is lowest (Laibson, Repetto and Tobacman, 2003; Gourinchas and Parker, 2002).

For intertemporal elasticity of substitution, we use $\sigma = 2$. Mehra and Prescott (1985) review microeconomic studies estimating $\sigma$ at about 1.5, and more recently Chetty (2006) estimates a bound of $\sigma \leq 2$ based on U.S. labor supply elasticities. In this model, the same parameter $\sigma$ governs both risk aversion and the intertemporal elasticity of substitution, since preferences are additively separable across both time and states. Calibration of risk aversion requires assumptions on the functional form of $u(\cdot)$. We will assume power utility, which is also known as CRRA utility for its constant relative risk aversion $\frac{cu''(c)}{u'(c)} = \sigma$.

3 Some Stylized Facts

Here we present several stylized facts in development that seem difficult to reconcile with the simple Euler equation approach above.

3.1 Unexploited High Returns

A number of studies show that farmers and entrepreneurs in developing countries face high marginal rates of return. Experiments giving extra cash or equipment to small firms find returns of 5% per month in Sri Lanka (De Mel, McKenzie and Woodruff, 2008), 15% per month in Ghana (Fafchamps, McKenzie, Quinn and Woodruff, 2014), and 20–33% per month in Mexico (McKenzie and Woodruff, 2008). Udry and Anagol (2006) compute a lower bound of 60% to the real return to capital in Ghana’s informal sector, using data on the prices and durability of new and used auto parts. Banerjee and Duflo (2005) conclude that “[t]he average of the marginal products of physical capital in India may be as low as 22%, though even reasonably large firms often have marginal products of 60%, or even 100%.”

Paying off debt is a high return investment when interest rates are high, so persistent high-interest borrowing is just as puzzling. Banerjee and Duflo (2005) cite several examples of high rates, such as 52% from moneylenders in rural Kerala and Tamil Nadu and 48% from urban finance corporations in urban India (Dasgupta, 1989), 5–7% per month for informal

\footnote{Note that even if utility were more curved at low levels of consumption it would still be difficult to simultaneously account for unexploited return investments and low demand for insurance.}
business finance in Thailand (Ghate, 1992), and 5% per month for manufacturing supply credit for black Africans in Kenya and Zimbabwe (Fafchamps, 2000). Ananth, Karlan and Mullainathan (2007) find that short-term borrowing for working capital at rates of 3–4% per month is common among small vendors surveyed in India and the Philippines.

Failing to invest at a high (risk-adjusted) rate is a failure of arbitrage if the agent has access to capital (credit or savings) at a lower rate. But even when investment must come from foregone consumption, such findings are surprising in light of the Euler equation. For example, calibrating with discount factor $\beta = .96$ and intertemporal elasticity $\sigma = 2$, an agent facing annual gross returns of $R = 1.80$ (5% per month) would have to experience consumption growth of 31% per year—a 15-fold increase every decade—to satisfy equation (1). Such rapid consumption growth is generally not seen in the data, to say the least.

Alternatively, calibrating to stable consumption requires a discount factor of $\beta = 1/R$, which with $R = 1.80$ is an implausibly small .56.

One possibility for why large investment opportunities could go unexploited is if there are non-convexities in returns, for example if there is some fixed minimum investment. However, unexploited high returns have been shown even for small investments for which such challenges are less relevant. Duflo, Kremer and Robinson (2011) find that farmers in Kenya fail to invest in fertilizer, despite a rate of return of 52–85% and its availability in small quantities. Beaman, Magruder and Robinson (2014) estimate that not having sufficient small change to break larger bills costs small firms in Kenya 5–8% of weekly profits. Kremer et al. (2011) estimate returns to inventory to small shopkeepers in Kenya to be 19% for avoiding stockouts and 100% to qualify for bulk purchasing discounts. Also, even if investments are lumpy, one could imagine contracts to deal with this.

According to the Euler equation, individuals facing very high marginal returns should see their consumption is growing rapidly. The existence of high returns without rapid consumption growth is therefore puzzling in light of the Euler equation.

### 3.2 Low Returns

A complementary puzzle is that many individuals who have high-return investment opportunities also hold lower return assets. The Euler equation dictates that the all assets should have the same risk-adjusted marginal returns, except those whose returns are too low even at a level of zero investment.

One commonly held asset is livestock, owned by 45% of rural Indian households as of 1999 according to the Rural Economic and Demographic Survey (REDS). Anagol, Etang and Karlan (2014) estimate that owning cattle earns households in India large negative median returns—of -293% for cows and -65% for buffaloes—with positive returns for buffaloes only.
under specifications that value labor as free or milk at higher than self-reported reported market value.

Low-interest bank accounts are also common. 88% of shopkeepers interviewed in Kremer et al. (2011) kept money in a savings account paying a few percentage points a year, despite average returns of 39% (with significant heterogeneity) for the surveyed type of inventory. Every household surveyed by Collins, Morduch, Rutherford and Ruthven (2009) maintained low-interest savings and high-interest loans at the same time. (They argue this behavior is for liquidity and cashflow purposes, as to secure access to loans for the future.) Some widely-used savings instruments, such the susu system in West Africa or deposit collectors in India, even have negative returns of -3% to -9% of deposits, as a fee for their labor-intensive deposit collection services (Opoku-Agyemang, 2012 Collins et al., 2009, Chapter 5; see also Rutherford, Mutesasira, Sampangi, Mugwanga, Kashangaki, Maximambali, Lwoga, Hulme et al., 1999; Ashraf, Karlan and Yin, 2005).

Of course, one possibility is that the people hold high-return assets because their payoff is strongly positively correlated with consumption, giving them high beta. The equity premium puzzle literature suggests investors in developed countries would need implausibly high rates of risk aversion, at least $\sigma > 30$, rather than the $\sigma < 2$ estimated from intertemporal calibrations, to explain why they do not hold high-returning equities, given equities’ small additional risk (Mehra and Prescott, 1985). We expect that in a classical framework similarly high rates of risk aversion would be needed to explain why households hold low and high return assets. As we discuss in below, such high rates of risk aversion would generate high demand for actuarially fair insurance, though demand for insurance is often low.

3.3 Low Insurance Takeup

Farmers in developing countries are exposed to significant weather risk, which can translate to consumption fluctuations with large negative welfare impacts (Ligon, 2010; Kazianga and Udry, 2006) as well as lower child health and education due to bad harvests (Morduch, 1995). In addition, standard theory is unambiguous that individuals should have a positive willingness to pay for reducing consumption variability. However, insurance schemes for low-income households typically achieve takeup of below 30% (Matul, Dalal, De Bock and Gelade, 2013).

This is not simply due to high costs or prices in the market, as many farmers turn down even actuarially fair insurance. In Ghana, Karlan et al. (2014) find a takeup of 40–50% for actuarially fair weather insurance. In India, Cole, Giné, Tobaeman, Townsend, Topalova and Vickery (2013) offer heavily discounted insurance to farmers, with an expected return of 70% (better than actuarially fair), and achieve takeup of less than 50%.
Health shocks form another serious risk, and health insurance smooths out the impact of this risk on the budget. However, health microinsurance schemes also see low takeup, such as 25% in Senegal and 19% in Kenya (Matul et al., 2013). Though, overall low usage of health insurance may be due to factors other than low intrinsic demand, such as market failures or substitutes such as government-subsidized healthcare.

### 3.4 Insurance Yields Large Increases in Investment

Once farmers receive insurance, particularly coverage against catastrophic risk, they tend to respond by increasing investment. Farmers offered rainfall insurance in India substituted from drought-tolerant to high-yield varieties of rice (Mobarak and Rosenzweig, 2012). Maize farmers in Ghana who received weather-linked insurance were able to find resources to make 13% higher cultivation expenditure, including fertilizer, land preparation, hired labor, while a cash grant had a much smaller effect on investment (Karlan et al., 2014).

Another case in which insurance causes a dramatic increase in investment is in migration. Bryan, Chowdhury and Mobarak (2014) find large average gains to seasonal migration to cities from a poor rural area of Bangladesh during a regular hunger season: on average, the migrant’s family increases food and non-food expenditures by 30–35% and caloric intake by 550–700 calories per person per day. Nevertheless, only 36% of control households migrated.

In this context, there are still some labor market opportunities in the rural area, albeit few and at a low wage, so migration does carry the risk of not finding work in the city and leaving one’s family even hungrier. When they experimentally offer a small incentive of approximately $11.50 to cover the round trip bus fare to the city plus a bit extra, which mitigated the downside risk, migration increased by 22 percentage points.

But, the authors calibrate that to justify such levels of non-migration and response to the experimental incentive, extremely high levels or risk aversion, on the order of $\sigma = 20$ in their midline specification, would be necessary. However, such a high $\sigma$ for a classical agent would also prescribe that the that the individual build up a large amount of precautionary savings for the famine season, which they do not observe.

### 4 Loss Aversion: Some Background

These puzzles of reconciling the classical framework with observed behavior have led to the application of ideas from behavioral economics. A leading approach is present-bias or myopia, in which agents behave for some decisions as if $\beta$ were very low, leading to low investment (Thaler and Shefrin, 1981; Laibson, 1997; O’Donoghue and Rabin, 1999;
Banerjee and Mullainathan, 2010). Other approaches focus on limits to cognitive ability and attention (Banerjee and Mullainathan, 2008; Mullainathan and Shafir, 2013), or awareness of returns as an explanation for under-investment and heterogeneity of returns (Conley and Udry, 2010; Hanna, Mullainathan and Schwartzstein, 2013).

Here, we take an alternative approach, based on the theory of loss aversion, advanced as part of prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991). Loss aversion holds that when individuals compare different options or make risky choices, they evaluate outcomes relative to a reference, and they weigh losses more heavily than gains.\(^5\)

Loss aversion has been applied to explain a number of real-world phenomena, ranging from labor supply and income targeting patterns of New York City taxicabs (Camerer, Babcock, Loewenstein and Thaler, 1997; see also Farber, 2005, 2008; Crawford and Meng, 2011) and Kenyan bicycle taxis (Dupas and Robinson, 2013), to design of incentives for teachers to improve student performance (Fryer, Levitt, List and Sadoff, 2012), to putting by professional golfers (Pope and Schweitzer, 2011).

A simple specification from K˝ oszegi and Rabin (2006) of prospect-theoretic preferences over an vector outcome \(\mathbf{x}\) and reference \(\mathbf{x}^r\) is one with a classical component and a “gain-loss” or reference-dependent component:

\[
U(X|X^r) = \sum_i u_i(x_i) + \sum_i \gamma_i v(u_i(x_i) - u_i(x_i^r))
\]

(5)

where \(x_i\) and \(x_i^r\) are the \(i\)th components of \(\mathbf{x}\) and \(\mathbf{x}^r\), \(u_i(\cdot)\) is classical valuation of commodity \(i\) (classical utility is assumed to be additively separable), the \(\gamma_i\) are preference weights, and \(v\) is a prospect-theoretic value function that takes the form

\[
v(z) = \lambda \chi[z < 0] = \begin{cases} 
z & \text{if } z \geq 0 \\
\lambda z & \text{if } z < 0 \end{cases}
\]

(6)

with loss aversion parameter \(\lambda > 1\). (\(\chi\) is a characteristic function that is 1 when its argument is true and 0 otherwise.)

With these preferences, the ratio of marginal utility from losses to gains for component \(i\),

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5Prospect theory also proposes that the value function exhibits diminishing sensitivity, meaning that \(v'(z)\) tends towards zero as \(|z|\) gets large, and that rather than true probability weights, individuals use nonlinear decision weights to evaluate risky prospects; we will not make use of either of these aspects.
evaluated at the reference (i.e. \( x_i = x_i^r \)) is

\[
\Lambda := \frac{dU}{dx_i} \bigg|_{x_i = x_i^r} - \frac{dU}{dx_i} \bigg|_{x_i = x_i^r} = \frac{1 + \gamma_i \lambda}{1 + \gamma_i}
\]

where the notation \( \frac{df}{dx} \bigg|_+ := \lim_{\tilde{x} \to x^+} \frac{df(\tilde{x})}{d\tilde{x}} \) indicates the right derivative, and \( \frac{df}{dx} \bigg|_- \) is the left derivative. For \( U \), they are not equal at \( x_i = x_i^r \), because \( U \) inherits \( v \)’s kink at 0.

Using lab experiments and surveys, \( \Lambda \) is commonly estimated to be in the range of 2–2.5 (Tversky and Kahneman, 1992; Ericson and Fuster, 2013).

Such preferences induce risk aversion even for small gambles.\(^6\) Consider \( G = [+g, \frac{1}{2}; -\ell, \frac{1}{2}] \), a 50-50 gamble that gives either a gain of \( g > 0 \) or a loss of \( \ell > 0 \). For simplicity, take preferences as single-dimensional over money with classical utility \( u(z) = z \) for illustrative purposes, and take reference to be 0, which is what the agent would get if she did not participate in the lottery.\(^7\) The expected utility from \( G \) is then \( E[U(G|0)] = \frac{1}{2}(x - \lambda y) \). The individual would be willing to make gamble if \( \frac{g}{\ell} \geq \Lambda \).

Aversion to small gambles poses a challenge to standard preferences over consumption or lifetime income. Rabin (2000) shows that specifications of utility can explain behavior over either small or large risks, but not both: the individual is either essentially risk-neutral over small gambles (low risk aversion) or unreasonably averse to large ones (high risk aversion). For example, any risk-aversion parameters that would make a classical agent turn down the gamble \([+$11, \frac{1}{2}; -$10, \frac{1}{2}] \) from any starting wealth level would also make her turn down \([+$g, \frac{1}{2}; -$100, \frac{1}{2}] \) for arbitrarily large \( g \).

Loss aversion also offers an explanation for the endowment effect, a tendency for individuals to assign higher values to items they already possess. Let \( X = (x_1, x_2) \in \{0, 1\}^2 \) denote discrete quantities of two commodities, and suppose preferences take the form of (5) with classical valuation \( u_i(z) = \alpha_i z \) for item \( i \), the initial endowment \((x_1^0, x_2^0)\) as the reference, and \( \gamma_i = \gamma_j = \gamma \). When faced with prices \((p_1, p_2)\), a classical agent would remain with her initial endowment only if \( \frac{p_1}{\alpha_1} \neq \frac{p_2}{\alpha_2} \); otherwise, she would trade away all of the item with the lower \( \frac{p_i}{\alpha_i} \) and spend all of her money on the other item. Her final bundle would not depend on her initial endowment.

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\(^6\)In particular, reference-dependent preferences induce “first-order risk aversion”, introduced by Segal and Spivak (1990), in which an agent’s willingness to pay to avoid a risk \( k \varepsilon \) for small constant \( k \) is of order \( k \). An expected utility maximizer with initial wealth \( W \) would be willing to pay a risk premium of \( \frac{1}{2} k^2 \text{Var} \left[ \varepsilon \right] \frac{u''(W)}{u'(W)} \), and so would have “second-order” risk aversion.

\(^7\)Equivalently, given initial wealth \( W \), the gamble leaves the agent with wealth of either \( W + g \) or \( W - \ell \), and she uses \( W \) as her reference.
A loss-averse agent would stick with the initial endowment if the relative price $\frac{p_1}{p_2}$ were in the interval $[\frac{\alpha p_1}{p_2}, \frac{\alpha p_2}{p_1}]$. The endowment effect predicts that people’s Willingness to Accept to sell an item exceeds their Willingness to Pay to buy it.  

Modeling a reference-dependent agent requires further specification of the environment than when dealing with classical preferences. First, the analyst must specify the framing or bracketing. It matters what the individual consider to be an “outcome”. In finance, this could mean evaluating performance stock-by-stock (narrow) vs. the whole portfolio (broad) (Barberis and Huang, 2008). Broad brackets tend to be less risk-averse, since gains in one dimension offset losses in another, reducing the probability of being in the loss domain. In consumption, this could mean treating each consumption good as a separate outcome (narrow), combining them all into aggregate consumption, or grouping together goods that satisfy similar hedonic desires (intermediate) (Kőszegi and Rabin, 2004).

Second, reference-dependent preferences also depend on the specification of the reference level. This specification of the varies in literature, such as the status quo (Kahneman and Tversky, 1979), lagged consumption (Chetty and Szeidl, 2010; Bowman, Minehart and Rabin, 1999), the risk-free rate of return (Barberis, Huang and Santos, 2001; Barberis and Huang, 2008), and rational expectations (Kőszegi and Rabin, 2006, 2007).

### 5 Investments by Loss-Averse Agents

#### 5.1 Model Setup

Loss-averse agents will tend to invest differently than classical agents, investing more in low-return assets while forgoing some risky opportunities for high return. Here, we present a model that illustrates how this could lead to a poverty trap for the loss-averse agent.  

Consider an agent faced with the same investment problem as in Section 2 but with loss-averse preferences over the bundle $\mathbf{x}_t = (c_t, I_t)$ of consumption $c_t$ and gross income $I_{it}$ from each asset $i$:

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8 This has been demonstrated in lab experiments, pioneered by Kahneman, Knetsch and Thaler (1990) with many variants since. There is an active literature on testing and explaining the endowment effect, and while some experimental specifications or market exposure can reduce or eliminate it, overall it is a robust finding; see Ericson and Fuster (2013) for a review.

9 The model has preferences of a form similar to Kőszegi and Rabin (2006, 2007) and includes reference-dependence with respect to consumption from their model, as well as reference-dependence with respect to investment income, bracketed narrowly as in the model of Barberis and Huang (2001).
\[
V(W_t, I_t | c^*_t, I^*_t) = \max_{c_t, \{ k_{it} \}} U(c_t, I_t | c^*_t, I^*_t) + \beta E_t \left[ V(W_{t+1}, I_{t+1} | c^*_{t+1}, I^*_{t+1}) \right] 
\]

\[
\text{s.t. } U(c_t, I_t | c^*_t, I^*_t) = u(c_t) + u'(\bar{c}_t)E_t \left[ \gamma_c v(c_t - c_t) + \sum_{i,j} \gamma_{ij} v(I_{it} - I^*_t) \right] 
\]

\[
c_t + \sum_i k_{it} \leq W_t \\
W_{t+1} := y_{t+1} + \sum_i I_{i,t+1} \\
I_{i,t+1} := f_i(k_{it}; \theta_{it}),
\]

where the i index assets and the j index reference points, \( u(\cdot) \) is classical utility from consumption, \( \bar{c}_t \) is a rational expectation of \( c_t \), the \( \gamma \) are weights, and the expectation operators are over both realizations of investments and references, in case these are stochastic, and \( \theta_{it} \) are the shocks. The agent’s utility \( U \) comes from classical utility from consumption \( u(c_t) \), plus reference-dependent utility from consumption \( c_t \) and income from each asset \( I_{it} \), where \( v(\cdot) \) is as in (6). The \( u'(\bar{c}_t) \) term converts these reference-dependent utility terms from units of consumption to utility.

For consumption, we specify that the reference equals its lagged realization, with \( c^*_{t+1} = c_t \). For investment, we allow multiple reference points \( I^*_t \), since comparisons to several alternatives might influence the agent’s thinking (Ericson and Fuster, 2013); for illustrative examples we will focus on one reference point at a time, though the effects we highlight would be present without this simplification. We consider two possible reference points: a lagged-realization reference, with \( I^*_t = I_{it} \), and comparison to a reference gross rate of return \( R^*_t \), with \( I^*_t = k_{it} \cdot R^*_t \). (For example, a reference rate of \( R^*_t = 1 \) means an investment considered loss-making if it returns less than was put in.) The latter specification aligns our preference specification closely with Barberis and Huang (2001, 2008, 2009) and related work on the disposition effect in financial investments.

Which reference is appropriate depends on the context and the psychology, such as how long agents have to get acclimated to their choices or how accustomed they have become to receiving investment returns.

5.2 First Order Conditions

The first order conditions for the agent’s choice of consumption and asset holdings are

\footnote{This ensures that \( \bar{c}_t = c_t \), but that in the agent’s optimization problem, \( \frac{d\bar{c}_t}{dc_t} \) is taken to be 0. In a macro model with identical agents, \( \bar{c}_t \) could be aggregate consumption, which equals \( c_t \) but which is out of the agent’s control (Barberis and Huang, 2008, 2009).}
\[
\max \left\{ \mu_{it}^+, \max_i \left\{ \mu_{it}^- \right\} \right\} = \mu_t^+ \leq \mu_t^- = \min_i \left\{ \mu_{it}^- \right\} \min_{k_t > 0} \left\{ \mu_{it}^- \right\},
\]
\[\text{with } \mu_{it}^+ := \frac{dU_t}{dc_t} \pm \]
\[\mu_{it}^- := \frac{dE_t [\beta V_{t+1}]}{dk_{it}} \pm \]

where \( \mu_{it}^+, \mu_{it}^-, k_t \) are the marginal values to increasing or decreasing \( c_t \) or \( k_t \) (expanded in Appendix A), and \( \mu_{it}^+ \) and \( \mu_{it}^- \) are Lagrange multipliers of the budget constraint, or equivalently right and left derivatives of indirect utility \( V(W_t; I_t, x_t) \) with respect to wealth \( W_t \).

Unlike in a classical problem, the agent might not be indifferent to all marginal uses of resources, and the marginal benefit of increasing consumption or an investment may not equal the marginal cost of decreasing it.\(^{11} \) Instead, the first order conditions are simply that transferring resources from one use to another must not improve welfare.

### 5.3 Implications of the Model

**Aversion to Risky Investments.** A loss-averse agent will be more reluctant to take on risky investments than a classical agent. Let \( i \) be an asset with stochastic returns, and make several simplifying assumptions. To focus on risk rather than endowment, assume the agent evaluates the return \( I_{i,t+1} \) against a single reference point \( I_{r,2} = R_s k_{it} \) with a constant rate of return, i.e. \( \gamma_{i1} = 0 \) and \( \gamma_{i2} > 0 \). To focus on risky investments, assume that \( \gamma_c = 0 \), which makes \( \mu_{it}^\pm = \mu_{ct}^\pm = u'(c_t) \), and that there is riskless savings asset \( s \) with returns \( R_s \) for which the agent has no reference-dependence, so the first order condition for consumption is

\[ R_s E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} \right] = 1 \]

as in the classical case. To highlight the differences from the classical model, assume that returns for \( i \) are independent of other risks and are a small part of the agent’s wealth, so that \( \theta_{it} \) (and hence \( f^*_it \)) can be approximated as independent of risk to future consumption \( c_{t+1} \). In order to simplify the condition for being in the loss domain, assume returns \( f_i(\cdot; \theta_{it}) \) are weakly concave in every state of the world \( \theta_t \). Then first order condition for \( k_i \) (if interior) is:

\(^{11}\)In particular, due to the kink in the value function \( v(\cdot) \) at zero, these two differ if there is positive probability that an outcome equals its reference.
$u'(c_t) = \beta E_t \left[ u'(c_{t+1}) \right] \left( E_t \left[ f'_{it}(k_{it}) \right] + \gamma_{i2} E_t \left[ f'_{it}(k_{it}) - R^*_i \right] \right) + (\lambda - 1) \gamma_{i2} E_t \left[ f'_{it}(k_{it}) - R^*_i \right] f_{it}(k_{it}) \leq R^*_i \; \mathbb{P}_t \left[ f_{it}(k_{it}) \leq R^*_i k_{it} \right] \right)$. 

A loss-averse agent gets lower marginal benefit from a risky investment than a classical agent due to the “$(\lambda - 1)$” term on the second line of (12). This term is unambiguously negative, and is more negative the greater are $\lambda$ (the loss aversion coefficient) and $\mathbb{P}_t \left[ f_{it}(k_{it}) \leq R^*_i k_{it} \right]$ (the probability that average returns fall short of the reference rate).\(^{12}\) It is also more negative the lower the expected marginal returns conditional on average returns falling short of the reference rate of return, which in particular makes investments that have a highly variable marginal return less attractive.

For a classical ($\gamma_{i2} = 0$) or reference-dependent but not loss-averse ($\lambda = 1$) agent, (12) with (11) reduces to $E_t \left[ f'_{it}(k_{it}) \right] = R_s$, equating the expected returns between the investment $i$ and savings $s$. That is, the agent behaves risk-neutrally with respect to asset $i$. Compared to this benchmark, in the case of a strictly concave return function, the “$(\lambda - 1)$” term in (12) reduces equilibrium $k_{it}$.

In the case of linear returns where $f_{it}(k_{it}) = R_{it} k_{it}$, (12) should be interpreted as an inequality: the agent will invest if the right hand side, which is is independent of $k_{it}$, exceeds $u'(c_t)$ on the left. Rather than reducing investment $k_{it}$ smoothly, loss aversion in this case means that the agent has a higher threshold for risky investments, and will turn down all but those with very high returns or with only a small downside risk.\(^{13}\)

To simplify equation (12), suppose that the investment either succeeds (with probability $p$) and has weakly concave return function $g(\cdot)$, or it fails completely, i.e. $f_{it}(k_{it}) = [g(k_{it}), p; 0, 1 - p]$. Then (12) with (11) reduces to

$$E_t \left[ f'_{it}(k_{it}) \right] = pg'(k_{it}) = \frac{R^*_i \gamma_{i2}(p + \lambda(1 - p)) + R_s}{1 + \gamma_{i2}}.$$ 

In this case, with $E_t \left[ f'_{it}(k_{it}) \right] f_{it}(k_{it}) \leq R^*_i k_{it}$ set to 0, the agent’s level of investment would be decreasing simply in the probability $1 - p$ of the investment failing (and of being in the

\(^{12}\)It is negative because $f'_{it}(k_{it}) - R^*_i \leq 0$ whenever $f_{it}(k_{it})/k_{it} \leq R^*_i$. By concavity of $f_{it}(\cdot)$, average product is always less than marginal product.

\(^{13}\)The model of K˝oszegi and Rabin (2007) features an “endowment effect for risk” in which agents who anticipate facing a risk are more willing to take it. Our model would replicate this effect, with agents being more willing to take on familiar than unfamiliar risks, if for example agents used for their reference point the stochastic process for the previous period’s income or returns—rather than its realization. For exposition, we have focused on a simpler specification, with deterministic reference points.
This result depends implicitly on narrow framing as well as loss aversion. If the individual faced many investments but bracketed them narrowly, she would evaluate them each with an expression like (12). But if she faced many identical independent investments and integrated the risk from them together into a compound asset, the diversification would generate an expected return with very little risk. So long as \( R_i \leq E[R_i] \), the 

\[
(\lambda - 1)
\]

term in (12) would vanish as the number of independent investments increased, so the extra distaste for risky investments from loss aversion would be small.14

**Low and Heterogeneous Returns.** Optimization for a classical agent will typically lead to a unique optimum. For loss averse agents, there may instead be a range of solutions (Kőszegi and Rabin, 2006). With loss aversion, the marginal benefits and costs to shifting along different margins need not be equalized at an optimum, there may be slack in the inequality first order conditions allowing the bundle to be adjusted without violating any of them.

Assume the agent faces no shocks and only invests in safe assets, so that there is perfect certainty in returns and consumption. For the reference, assume that \( \gamma_{i2} = 0 \) for all assets \( i \), so that the agent uses the previous period’s asset-\( i \) income for a reference, i.e. \( R_{i,t+1} = R_{it} \), and that the weight \( \gamma_{i1} \) is equal to the same constant \( \gamma_1 \) for all \( i \). A steady-state is described by the budget

\[
c_{ss} = y + \sum_i f_i(k_{i,ss}) - k_{i,ss}.
\]

where investment is such that all asset returns fall in a range:

\[
\frac{1 + \gamma_c}{1 + \gamma_c + \lambda \gamma_1} \leq \beta f_i(k_{i,ss}) \leq \frac{1 + \lambda \gamma_c}{1 + \lambda \gamma_c + \gamma_1}.
\]

(13)

See Appendix A.1 for derivation. The upper bound for \( \beta f_i(k_{i,ss}) \) is the condition that it is unattractive to increase \( k_i \) and decrease \( c \), and the lower bound is that it is unattractive to increase \( c \) and decrease \( k_i \). (A similar, but more complicated inequality condition holds with stochastic investments for which \( \gamma_{i1} > 0 \).)

14By the law of large numbers, the average return would be distributed approximately normally about a mean of \( E[R_i] \), with variance decreasing in the number of independent investments, so either \( \Pr_i[R_i \leq R_i^*] \) (if \( R_i^* < E[R_i] \)) or \( \Pr_i[R_i - R_i^* | R_i \leq R_i^*] \) (if \( R_i^* = E[R_i] \)) would go to zero.
Rather than setting returns equal for all assets as in the classical case, the agent’s investment behavior may make some higher than others. If we assume that $\gamma_c = \gamma_1 \equiv \gamma$, then the ratio of gross returns between assets can be as great as $1 + \frac{(\lambda - 1)\gamma}{1 + \gamma}$.

As with equation (12) for risky investments, for assets with constant returns $f'_i = R_i$, equation (13) should be interpreted as the range of $\beta R_i$ that is consistent with steady-state investment: too low and the agent won’t invest, too high and her wealth will grow in every period.

The agent may therefore hold assets with low marginal returns merely because they were already in her endowment. Although disinvesting could free up resources in higher-return savings vehicles (or even higher-return risky investments), doing this would be treated as a painful loss. But if she had not started with them, the gain from investing would not be great enough to induce her to invest either.

Compare this to a classical agent, for whom a (non stochastic) steady-state requires $\beta f'_i(k_i) = 1$ for all assets, a simple condition that is independent of previous asset-holdings.

### 6 Applications of the Loss Averse Model

As shown in the previous section, loss aversion can lead agents to invest less in high-return risky investment opportunities and more in low-return safe ones with which they are endowed. Here we show how it can shed further insight on other behavioral regularities.

#### 6.1 Poverty Traps and Stickiness

Models of poverty traps—in which agents who begin with low income or wealth remain poor—are typically based on frictions and nonconvexities, such as in nutrition (Dasgupta and Ray, 1986), education (Galor and Zeira, 1993), or capital (Banerjee and Newman, 1993). Banerjee and Mullainathan (2010) generate a similar result with a behaviorally-motivated regressive “temptation tax” on savings. Such models typically lead to a discontinuity in the relationship between the agent’s current and future endowment (e.g. of human capital), with two extreme stable steady-state levels and an unstable steady-state.

---

$^{15}$Although the interval for $\beta f'$ in (13) is below 1—reflecting that the agent gets utility from investment gains that a classical agent does not—this does not necessarily mean that a loss-averse agent always invests more than a classical agent. Rather, the appropriate comparison for a classical agent with discount factor $\beta$ might be a loss-averse one with a lower $\beta$, since the tradeoff between consuming in different time periods is not just discounting, but gains or losses from reference-dependence according to (13). An alternative comparison for a loss-averse agent might be one with reference-dependent preference without, i.e. with loss aversion coefficient $\lambda = 1$. 

---

16
in between. Such phenomena can also arise out of credit market imperfections, which make initial wealth endowment more important (Banerjee, 2003).

Like a classical poverty trap, loss aversion can limit economic mobility, but with a different dynamic process. Rather than creating two absorbing “poor” and “rich” steady-states, loss aversion leads to a range of stable asset holdings that are consistent with optimization, as in (13), leading to a possible stickiness of asset endowment. Rather than evolving to a bimodal distribution of wealth levels, loss aversion suggests a more continuous range. This result is consistent with the findings of Fafchamps et al. (2014), comparing firms that receive a grant of capital in kind or in cash, that the capital sticks in its initial form those receiving cash used more of it for consumption, and only those who received in-kind gifts grew their business profits.

Under a classical poverty trap, receiving a capital injection can lead to higher sustained income (see Blattman, Fiala and Martinez (2014) for a recent example). An agent who receives an asset with low returns, perhaps land, could sell the asset and use the resources to finance a higher-return investment, increasing her family’s long-term wealth or consumption. Under loss aversion, she may remain with the low-return asset, since once her reference point adjusts and the asset enters into her endowment, she will be reluctant to sell it. Or, loss aversion on consumption might lead the asset to be gradually sold to protect against negative consumption shocks.

These possibilities offer an explanation for results of 19th century land lotteries in the U.S. The sons of those who win the lotteries do not have higher human capital than those who do not (Bleakley and Ferrie, 2013a), and while for those initially above median wealth, receiving land led to higher family wealth 18 years later of the approximate magnitude of the value land, below the median there was no change in the distribution of wealth (Bleakley and Ferrie, 2013b). A loss aversion interpretation would be a kind of poverty trap in agriculture: land received was not parlayed into higher-return assets, and for poorer recipients, the land was eventually sold away to finance consumption.

Asset stickiness could have long-term and aggregate impacts on markets. Bleakley and Ferrie (2014) also find that the initial land parcel size was highly persistent, only dissipating 150 years after the land lottery, and that the pattern of land ownership depressed land values by 20% in the late 19th century. Joining parcels requires the cooperation of several landowners, so it could be inhibited by even a fraction of owners valuing them significantly more than potential buyers out of loss aversion.
6.2 Up-Front Costs

Depending on how agents frame their costs and benefits, loss aversion could lead agents to be particularly sensitive to up-front costs. In particular, in an investment with a long time horizon, if agents treat initial costs as a lost, separating them (at least initially) from later gains, then mitigating such costs may encourage more investment.

The experimental treatment by Bryan et al. (2014) can be viewed in this light: by providing households with the finance to cover the initial investment to migrate (namely transportation to the city), they increased migration by 22 percentage points.

Duflo et al. (2011) achieved a similar effect, increasing fertilizer usage from 34% to 45%, by offering farmers a special offer to purchase fertilizer just after they received payment for their harvest, rather than at the typical time of fertilizer purchase, at planting. Loss aversion would predict this result if farmers treated giving up some of their new harvest cash as foregone gain, but giving up cash months later—when it had already entered into their endowment—as a loss.

Reducing up-front losses or costs has proved effective in other contexts. It is one of the key principles the Save More Tomorrow program that has increased retirement savings among employees in the U.S. (Thaler and Benartzi, 2004).

6.3 Agricultural Insurance and Investment

Loss aversion helps explain the two stylized facts mentioned about agricultural insurance in the developing world: low takeup even at actuarially fair prices, and increased investment after receiving insurance. In contrast to unrealized returns, these patterns are not necessarily inconsistent with classical theory, but rather loss aversion provides a complementary explanation.

To explain low takeup with a classical model requires that the agent be concerned about basis risk: that she will have low income but the insurance instrument (e.g. rainfall) will not be triggered. It also requires strong assumptions about the shape of the utility function, namely that the coefficient of risk aversion is sharply decreasing in income, so that the state of the world in which the agent paid for insurance and also had a crop failure has sufficiently low utility as to overwhelm the beneficial effects of the insurance when it does pay out correctly.

Similarly, classical preferences can deliver that insurance leads to greater productive investment (see Appendix A of Karlan et al. (2014) for a simple model), though only if the market is not offering insurance.
On the other hand, loss aversion can augment the discouraging effect of basis risk, particularly if the farmer’s reference in the bad state is that she will at least have the minimum level of consumption provided by the insurance contract: in this case, a failure to pay out would be coded as a loss.

Loss aversion can also make an individual more willing to invest once he has insurance, since it reduces the marginal contribution of investing to landing the individual in the loss domain.

6.4 Credit Contract Design to Incentivize the Loss-Averse to Invest

If loss aversion leads to low investment in normal circumstances, there may be arrangements that can incentivize a loss averse agent to invest more. As noted above, the provision of insurance is one possibility.

Another is judicious designs of credit contracts. Theory predicts that loss averse agents will be more attracted to loans that use a new asset for collateral, as in a mortgage, than those that use an asset the agent already owns. A default event is less painful—and the expected benefit of the loan is higher—if parting with the collateral is not coded as a loss.

Consider a loan issued in period 0, to be paid back in period $T$, that is used to purchase a quantity $L$ of a new asset $n$. It requires a total collateral of $C$, and its terms specify a split into $C_n$ of the new asset and $C_o$ of some (old) asset $o$ that the agent already owns, with $C_n + C_o = C$ and $C_n \leq L$. If the agent defaults, she relinquishes the collateral.\(^{16}\) Let $\hat{k}_i$ be the ownership of asset $i \in \{n,o\}$ just after the loan is issued. For simplicity, suppose the agent previously did not own the new asset, meaning that $k_{n,-1} = 0$ but $\hat{k}_n = L$, and that her ownership of $o$ does not change after the loan, meaning that $\hat{k}_o = k_{o,-1}$.

For simplicity, assume returns are linear with $f_i(k_i) = R_i k_i$. Suppose further that in the absence of default, the agent has no reason to adjust ownership levels from the $\hat{k}_i$. Then if the agent does not default ($ND$), her income from $i \in \{n,o\}$ in each period $t > 0$ is $I_{it}(ND) = R_i \hat{k}_i$, while if she does default ($D$) in $T$, then for $t \geq T$ $k_{it}$ is reduced to $\hat{k}_i - C_i$ so for $t > T$ income from $i$ is $I_{it}(ND) = R_i (\hat{k}_i - C_i)$.

Defaulting reduces utility through two channels. First, as in a classical model, it reduces income and hence consumption; this cost does not depend on how collateral is split up between assets $n$ and $o$.\(^{17}\) Second, for a loss-averse agent, it also reduces the reference-dependent component of utility. Suppose that $\gamma_{i,1} > 0$ but $\gamma_{i,2} = 0$, so that this component

\(^{16}\)We focus on nonstrategic default, though the model can shed light on incentives to pay on time as well.

\(^{17}\)For simplicity, here a default event simply occurs or does not, rather than coming from some external trigger like a consumption or health shock; in the latter case, default would mean paying back the loan with collateral rather than cash, and its classical costs would come from transaction costs or a divergence between private and market value of the collateral.
comparative levels of income (rather than rates of return). Then the difference in this component between default and no default, which is realized in \(T + 1\), one period after the default event, is (with the appropriate coefficient \(u'(\tilde{c}_{T+1})\) to convert to utility):

\[
\sum_{i \in \{n, o\}} \gamma_{n1} \left[ v\left( \tilde{I}_n - \tilde{I}^{r,1}_{i,T+1} \right) \right]_{\tilde{I}_i = I_{i,T+1}(\text{ND})} = \sum_{i \in \{n, o\}} \gamma_{n1} \left[ v\left( \tilde{I}_o - \tilde{I}^{r,1}_{i,T+1} \right) \right]_{\tilde{I}_i = R_i(\hat{k}_i - C_i)} \quad , (14)
\]

with notation \([f(x)]_{x=b}^{x=a} := f(b) - f(a)\). The value of this component depends on the references \(\tilde{I}^{r,1}_{i,T+1}\), so it depends on the reference-setting rule and the timing of the loan. We fix the rule as one-period lagged-realization proposed in Section 5.1, in which \(I^{r,1}_{it} = I_{i,t-1}\), and we consider two cases of default: a short-term loan with default (or repayment) at \(T = 0\), and long-term loan with \(T > 0\).

In the case of \(T = 0\), the reference at \(T + 1 = 1\) is based on ownership before the loan: \(I^{r,1}_{n,1} = I_{n,0} = R_n k_{n,-1} = 0\) and \(I^{r,1}_{o,1} = I_{o,0} = R_o k_{o,-1} = R_o \hat{k}_o\). The reference-dependent component of disutility of defaulting (14) is therefore:

\[
\gamma_{n1} \left[ v\left( \tilde{I}_n - 0 \right) \right]_{\tilde{I}_n = R_n(\hat{k}_n - C_n)} + \gamma_{o1} \left[ v\left( \tilde{I}_o - R_o \hat{k}_o \right) \right]_{\tilde{I}_o = R_o \hat{k}_o} = -\gamma_{n1} R_n C_n - \lambda \gamma_{o1} R_o C_o.
\]

The \(\lambda\) on the \(o\) term indicates that giving up \(C_o\) of \(o\) is treated as a loss, while giving up \(C_n\) of \(n\) is considered a foregone gain, since the new asset has not yet entered the agent’s reference. Thus, default will be more painful the more of the collateral is in \(o\).

However, in the case of \(T > 0\), the references in \(T + 1\) adjust to reflect asset ownership after the loan: \(I^{r,1}_{i,T+1} = I_{i,T} = R_i \hat{k}_{i,T-1} = R_i \hat{k}_i\) for both \(i = n\) and \(i = o\). Then the reference-dependent component of disutility of defaulting (14) is:

\[
\gamma_{n1} \left[ v\left( \tilde{I}_n - R_n \hat{k}_n \right) \right]_{\tilde{I}_n = R_n(\hat{k}_n - C_n)} + \gamma_{o1} \left[ v\left( \tilde{I}_o - R_o \hat{k}_o \right) \right]_{\tilde{I}_o = R_o \hat{k}_o} = -\gamma_{n1} R_n C_n - \lambda \gamma_{o1} R_o C_o. \quad (15)
\]

Here, if the weight on reference-dependence is equal (\(\gamma_n = \gamma_{ox}\)) and the returns are also equal (\(R_n = R_o\)), then the agent is indifferent to the form of collateral, since the two \(\lambda\) coefficients in (15) indicate that giving up either asset would be coded as a loss: the higher
income level \( I_i = R_i \hat{k}_i \) will have become the reference.

The first case, in which losing the new asset is not so painful, is relevant whenever the agent uses the earlier reference \( I^r_n = 0 \) when considering default. This could apply even for a long-term loan \((T > 0)\) if there were a different reference-setting rule. For example, when considering ex ante at \( t = 0 \) whether to take the loan, the agent might treat \( I^r_{i,T+1} = 0 \). This could be interpreted as either projection bias about future reference-dependent utility (Loewenstein, O’Donoghue and Rabin, 2003), or intertemporal conflict in preferences (O’Donoghue and Rabin, 1999). The first interpretation represents a mistake in which the agent thinks she is acting in her best interest, but in fact sets herself down a path in which she might face a default that is costlier than she anticipated; in this case, allowing the asset purchased with the loan to be used for its own collateral would reduce welfare. In the second interpretation, while the period-\( T \) self, facing the possibility of default, might regret having taken up the loan, the period-0 self would not have any regrets.\(^{18}\) Here, a the welfare judgment is ambiguous (see Section 7.2).

Regardless of the type of collateral and reference used when considering the loan ex ante, if while paying it off, the reference for \( n \) moves from \( R_n \hat{k}_{n,-1} \) to \( I^r_n = R_n \hat{k}_n \), then losing the collateral and reducing income by \( R_n C_n \) will loom as a loss, so the agent will willing to work hard or sacrifice consumption to avoid it.\(^{19}\) So, using the new asset for its own collateral could both make borrowing more attractive than using a familiar asset while making the agent just work as hard to not lose it.

## 7 Concluding Thoughts

### 7.1 Loss Aversion and Industrial Structure

Experiments of the endowment effect find that context, framing, and individual characteristics all can influence whether individuals (Ericson and Fuster, 2013; List, 2003). There is evidence of interpersonal heterogeneity in loss aversion.

For instance, Benjamin, Brown and Shapiro (2013) find that for high school students, math scores (and even decade-old elementary-school grades) predict small-stakes risk aversion and short-term discounting in lab experiments. Similarly, Frederick (2005) argues for a connection between intelligence and certain behavioral tendencies, presenting data that low performance on a simple cognitive tests correlates with small-stakes risk aversion and

\(^{18}\)In this situation, it would not matter if the earlier self were sophisticated (understanding the conflict in preference) or naive, since in effect taking the loan is a commitment device, obliging the later self to work hard or sacrifice in order to avoid a painful default event.

\(^{19}\)To the extent to which the reference is less entrenched (smaller \( \gamma_i \)) for the novel asset \( n \) than for the old one \( o \), the effect will be even stronger since the agent will be less affected by changes in income from \( n \) than from \( o \).
impatience. Various theories connect cognitive skills to behavioral biases, such as additional reflection before impulsive decisions, consistent with a “two systems” theory in psychology (Kahneman, 2011).

One perspective on behavioral biases is that the division of labor will mitigate their effects in the market. As Becker (quoted in Stewart, 2005) asserts: “It doesn’t matter if 90 percent of people can’t do the complex analysis required to calculate probabilities. The 10 percent of people who can will end up in the jobs where it’s required.” Kremer et al. (2013) show that behavioral factors are indeed relevant for firms’ investment decisions in a developing country context: math and Raven’s matrices scores predict higher inventories in a context in which the return on inventory is high. Similarly, when presented with computed amounts of sales lose due to inadequate change, 83% of firm surveyed by Beaman et al. (2014) reported that it was more than they anticipated, and 85% said they planned to adjust their behavior to get more change regularly. Additionally, Kremer et al. (2011) find that firms because less likely likely to report stockouts the longer they have been monitored, suggesting that the surveys made the lost income more salient.

Moreover, there are reasons why this might apply even less in developing economies than in developed ones, due to their industrial structure.

The tendency for experienced and skilled individuals to make investment choices may be lower in developing countries, because which firms have fewer employee and self-employment more common (Gollin, 2008; Poschke, 2011). In the Kremer et al. (2011) sample, for example, retail shops are are typically owner-operated, with owners managing just one shop. To the extent to which capital or family ties determine management responsibilities, rather than ability or competitive hiring, more decisionmakers are likely to misallocate capital due to behavioral biases.

7.2 Welfare

Are loss-aversion and small-stakes risk aversion “mistakes”? Kahneman, Wakker and Sarin (1997) distinguish between decision utility and experienced utility, and argue that decision utility is often based on biased memories of experienced utility, making the latter closer to welfare. In his recent book, Kahneman (2011) advocates that his readers bracket widely in order to worry less and live happier lives. More specifically, Kermer, Driver-Linn, Wilson and Gilbert (2006) argue based on hedonic surveys that loss aversion is a form of projection bias: individuals overestimate how much distress losses will cause them.

According to this view, then, missed opportunities due to loss aversion do not simply make people poorer, but worse off.

An alternate view is that small-stakes risk aversion and related phenomena reveal pref-
erences and do not need to be “corrected”. Perhaps, for example, low migration found by Bryan et al. (2014) is because the migrating is simply unpleasant. Though, under that view, it is surprising that a small amount of money can induce migration not just contemporaneously, but in future years after the incentive is removed.

While we have argued that loss aversion may be an important factors to explaining patterns of investment, insurance, and asset ownership, the ultimate impact of these depends the interpretation of these behavioral tendencies.
A Mathematical Appendix

A.1 Marginal Benefits and Costs in the First Order Conditions of Section 5.2

The marginal benefits and costs to first order condition (8) for the loss-averse investor’s problem (7) are

\[
\mu^+_t := \frac{dU_t}{dc_t} \bigg|_+ = u'(c_t) \left( 1 + \gamma c E \left[ \lambda I_{t+1}^r \right] \right) 
\]

\[
\mu^-_t := \frac{dU_t}{dc_t} \bigg|_- = u'(c_t) \left( 1 + \gamma c E \left[ \lambda I_{t+1}^r \right] \right) 
\]

\[
\mu^+_i := \frac{dE_t[\beta V_{t+1}]}{dk_{it}} \bigg|_+ = \beta E_t \left[ \frac{dI_{i,t+1}}{dk_{it}} \mu^+_t + \gamma i u'(c_{t+1}) \frac{d}{dk_{i+1}} \bigg|_+ v \left( I_{i,t+1} - I_{i,t+1}^r \right) \right] 
\]

\[
\mu^-_i := \frac{dE_t[\beta V_{t+1}]}{dk_{it}} \bigg|_- = \beta E_t \left[ \frac{dI_{i,t+1}}{dk_{it}} \mu^-_t + \gamma i u'(c_{t+1}) \frac{d}{dk_{i+1}} \bigg|_- v \left( I_{i,t+1} - I_{i,t+1}^r \right) \right] . 
\]

The expectation in \( \frac{dU_t}{dc_t} \bigg| \pm (16)\) is over \( c^r_t \), in case this is stochastic (in contrast, \( c_t \) can not be stochastic because the agent chooses it after \( \theta_t \) is revealed), while the expectation in \( \frac{dE_t[\beta V_{t+1}]}{dk_{it}} \bigg| \pm (18)\) is over both income realization and reference. In (18)–(19), unless the probability is zero that the outcome \( I_{i,t+1} \) equals the reference \( I_{i,t+1}^r \), the contribution of the reference-dependence term \( v(\cdot) \) depends on the direction of the deviation \( dk_i \) and on the specification of reference, according to

\[
\frac{d}{dk_{it}} \bigg| \pm v \left( I_{i,t+1} - I_{i,t+1}^r \right) = \lambda \chi[I_{i,t+1} \pm dI_{i,t+1} < I_{i,t+1}^r \pm dI_{i,t+1}^r] 
\]

\[
= \begin{cases} 
1 & \text{if } I_{i,t} - I_{i,t}^r > 0 \\
1 & \text{if } I_{i,t} - I_{i,t}^r = 0 \text{ and } \pm \left( \frac{dI_{i,t}}{dk_{it}} - \frac{dI_{i,t}^r}{dk_{it}} \right) \geq 0 \\
\lambda & \text{if } I_{i,t} - I_{i,t}^r < 0 \\
\lambda & \text{if } I_{i,t} - I_{i,t}^r = 0 \text{ and } \pm \left( \frac{dI_{i,t}}{dk_{it}} - \frac{dI_{i,t}^r}{dk_{it}} \right) \leq 0 
\end{cases} . 
\]

In particular, if the reference income is specified to be a fixed amount, then \( \frac{dI_{i,t+1}^r}{dk_{it}} = 0 \), so
while if there is a reference rate of return rather than a reference income level, so that $I_{t+1}^r = R^r k_{it}$ and $dI_{t+1}^r/dk_{it} = R^r$, then if returns $f_{it}(-)$ are concave,\(^{20}\)

$$
\begin{align*}
\frac{dE_t[\beta V_{t+1}]}{dk_{it}} &= \beta E_t \left[ f'_{it}(k_{it}) \left( \mu_{t+1}^+ + \gamma_i u'(c_{t+1}) \lambda \chi(f_{it}(k_{it}) < I_{t+1}^r) \right) \right], \\
\frac{dE_t[\beta V_{t+1}]}{dk_{it}} &= \beta E_t \left[ f'_{it}(k_{it}) \left( \mu_{t+1}^- + \gamma_i u'(c_{t+1}) \lambda \chi(f_{it}(k_{it}) \leq I_{t+1}^r) \right) \right].
\end{align*}
$$

A.2 Steady-State with One Asset

With consumption and one savings asset $s$, and with $\gamma_{s,1} > 0$ and $\gamma_{s,2} = 0$ so that the reference for savings is simply the steady-state income level $I_{s,ss}^1 = f_s(s_{ss})$, the first order conditions in in steady-state are

$$\max \{ \mu_{c,ss}^+, \mu_{s,ss}^+ \} = \mu_{ss}^+ = \mu_{ss}^- = \min \{ \mu_{c,ss}^-, \mu_{s,ss}^- \}.$$

Whether a steady-state is supported, and if so which marginal costs and benefits bind, depends on where $\beta f_s^s$ falls with respect to four thresholds: $\frac{1+\gamma_c}{1+\gamma_c+\gamma_{s,1}} < \frac{1+\gamma_s}{1+\gamma_c+\gamma_{s,1}} < \frac{1+\gamma_s}{1+\gamma_c+\gamma_{s,1}}$. To summarize:

- $\mu_{ss}^+ = \mu_{s,ss}^+ = \frac{\beta f_s^s(s_{ss})}{\alpha f_s^s(s_{ss})} u'(c_{ss}) \gamma_{s,1}$ if $\beta f_s^s(s_{ss}) \geq \frac{1+\gamma_c}{1+\gamma_c+\gamma_{s,1}}$.
- $\mu_{ss}^+ = \mu_{c,ss}^+ = u'(c_{ss})(1+\gamma_c)$ if $\beta f_s^s(s_{ss}) \leq \frac{1+\gamma_c}{1+\gamma_c+\gamma_{s,1}}$.
- $\mu_{ss}^- = \mu_{c,ss}^- = u'(c_{ss})(1+\gamma_c)$ if $\beta f_s^s(s_{ss}) \geq \frac{1+\gamma_s}{1+\gamma_c+\gamma_{s,1}}$.
- $\mu_{ss}^- = \mu_{s,ss}^- = \frac{\beta f_s^s(s_{ss})}{\alpha f_s^s(s_{ss})} u'(c_{ss}) \gamma_{s,1}$ if $\beta f_s^s(s_{ss}) \leq \frac{1+\gamma_s}{1+\gamma_c+\gamma_{s,1}}$.
- If $\beta f_s^s(s) > \frac{1+\gamma_c}{1+\gamma_c+\gamma_{s,1}}$ for all $s$, then there is no steady-state, and savings grows without bound. This can occur if, for instance, $f_s^s = R_s$ is fixed and high.
- If $\beta f_s^s(s) < \frac{1+\gamma_s}{1+\gamma_c+\gamma_{s,1}}$ for all $s$, then there is no steady-state, and savings shrinks to zero. This can occur if, for instance, $f_s^s = R_s$ is fixed and low.

\(^{20}\) Concavity ensures that $f_{it}(k_{it}) = R^r k_{it} \Rightarrow f'_{it}(k_{it}) < R^r$, marginal product is below average product.
Note that since \( \frac{1 + \gamma_c}{1 + \gamma_c + \gamma_s} > \frac{1 + \lambda \gamma_c}{1 + \lambda \gamma_c + \lambda \gamma_s} \), so \( \mu_{ss}^+ = \mu_{c,ss}^+ \) and \( \mu_{ss}^- = \mu_{c,ss}^- \) can occur simultaneously, while \( \mu_{ss}^+ = \mu_{c,ss}^+ \) and \( \mu_{ss}^- = \mu_{c,ss}^- \) cannot.
References


