The Demographic Transition and the Position of Women: A Marriage Market Perspective

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Abstract

We present international evidence on the marriage market implications of cohort size growth, and set out a theoretical model of how marriage markets adjust to imbalances. Since men marry younger women, secular growth in cohort size in the second phase of the demographic transition worsens the position of women. This effect has been substantial in many Asian countries earlier, and continues to be true in sub-Saharan Africa. With fertility decline, cohort are now shrinking in East Asia, improves the position of women. These demographic trends can explain the increase and spread in dowries in South Asia and the persistence of polygyny in sub-Saharan Africa. We show that the age gap at marriage will *not* adjust in order to equilibrate the marriage market in response to persistent imbalances, even though it accommodates transitory shocks.

JEL Categories: J12, J13, J16

Keywords: sex ratio, marriage markets, marriage squeeze.

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1 Introduction

The demographic transition has major implications for the position of women within the family, and in society. Fertility declines as parents trade "quantity for quality" in their children, and investment in human capital becomes increasingly important. Doepke and Tertilt (2009) argue that increasing requirements for human capital investments is what made men want to relinquish their monopoly on property rights, and grant them to women. Fernandez (2010) sets out a model where the decline in fertility plays an important role in this, and uses data from the state-wise variation in the granting of property rights in the US to test this model. While the broad brush picture of the world in the last two centuries indicates a dramatic expansion in womens' rights, within marriage and more generally in society, this trend has not been as universal or comprehensive as might be expected. The developing world offers a more nuanced picture. For example, in India, concerns have been raised about "missing women", and the trend of rising dowries, and the consequent financial "burden" that daughters impose on their parents. In sub-Saharan Africa, polygamy continues to be an important phenomenon despite modernization and economic development.

The paper focuses on one implication of the demographic transition, for the marriage market and for the consequent position of women. This appears not to have been previously been appreciated, although demographers have pointed out some aspects of this picture. Our focus is on the rate at which marriage cohorts grow, in the different phases of the demographic transition.[1] Given that men marry younger women, systematic growth in cohort sizes implies that each cohort of men is matched with a larger cohort of women, giving rise to a *marriage squeeze* on women, i.e. their excess supply.[2] Similarly, systematic decline in cohort sizes imply a reverse marriage squeeze, where men are in excess supply. The consequent marriage market effects are substantial. Deferring the detailed evidence to section II, the stylized facts are as follows (see Fig. 1). In phase I, cohort sizes are basically static or growing very slowly. In phase II, with the decline in mortality, especially infant mortality, cohorts grow rapidly, at 2-3% per annum in many developing countries. With an age gap at marriage of 4-5 years, this translates into a 8-15% increase in the effective supply of women, as compared to phase

[1] While the demographic transition is normally phrased in terms of population growth, the relevant variable for the marriage market is the rate of growth in marriage cohort size, which differs somewhat from population growth.Empirically, the two measure can diverge quite substantially.

[2] Demographers use the term marriage squeeze (see Akers, 1977; Schoen, 1984) to denote a marriage market imbalance, due to the effective excess of women or men. This may arise due to shocks to the marriage market sex ratios, e.g. due to wars or due to variations in the sex ratio at birth, or transitory shocks to cohort size, due to baby booms or famines, give rise to imbalances. Bergstrom and Lam (1989a) demonstrate that there are large variations in cohort size in 19th century Sweden. In the Chinese famine of 1959-61, cohort sizes fell by 75% (Brandt, Siow and Vogel, 2008) The focus of the present paper is on the effects of systematic growth or decline in cohort sizes.

I. In phase III, cohort size growth becomes either zero or significantly negative, at -1 to -2% per annum, implying an increase in the supply of men of 4-10% as compared to phase I. As compared to phase II, the change can be as much as 25%.

These changes in the effective excess supply of women have major implications for the balance of power between the sexes and for the allocation of resources within the household. Angrist (2002) and Chiappori, Fortin and Lacroix (2001) find that important effects on female labor supply and household allocation even for significantly smaller changes in marriage market balance. In the Indian context, demographers such as Bhatt and Halli (1999) have argued that the marriage squeeze is responsible for the deterioration of the position of women in India, and replacement of the institution of bride price in many regions and communities by dowries (payment from the bride's family to the groom). [3] Rao (1993) analyzes data on dowries from a sample of Indian villages and attributes the increase in dowries in India to the marriage squeeze.

Our focus in the paper is on marriage market adjustment mechanisms. One obvious mechanism is polygamy – i.e. polygyny in phase II and polyandry in phase III. This can be an important factor, especially in societies where legal and social sanctions against polygyny are absent. For example, in sub Saharan Africa, cohort size growth continues to be substantial, implying an significant excess of women in the marriage market relative to the number of men. This could be an important factor explaining the surprising persistence of polygamy in the face of modernization. Similarly, in Punjab in India, the persistent excess supply of women has been a historical feature, due to male biased sex ratios. Polyandry has been historically prevalent in the Punjab, but has declined in the last century. Our argument suggests that cohort size growth may have more than offset male biased sex ratios, and that this may have played an important role. Polygamy may not, however, be very appealing for the more abundant sex, and in this case, it utility consequences for the abundant sex may not be very different from non-marriage.

If we abstract from polygyny, the marriage market is an assignment market, in the sense of Shapley and Shubik (1972) since a man can be matched to at most one woman. The key potential margin of adjustment is via the age gap at marriage. If the age gap at marriage were to adjust in response to marriage market imbalances, they could be reduced, if not eliminated. Indeed, a reduction in the age gap reduces both the excess of women in phase II and the excess of men in phase III. While the age gap at marriage tends to fall in the process of development, as women become more educated, our question is whether, the age

[3] Anderson (2007) examines the time path of dowries in response to an one-off increase in the number of women. She finds that that dowries rise and then fall. This analysis is less relevant to the question of sustained population growth.

gap falls *endogenously*, in response to marriage market imbalance. Bergstrom and Lam (1989a, 1989b) and Brandt, Siow and Vogel (2008) have examined the effects of temporary shocks to birth cohort size, using data from Sweden and from the Chinese famine of 1959-61 respectively. These papers use a transferable utility assignment model in the tradition of Becker (1981), and find that marriage markets display considerable flexibility – the age gap at marriage adjusts in order to accommodate large shocks to cohort size. The empirical findings of Brandt et al. are particularly noteworthy, given that the Chinese famine reduced cohort sizes by 75%.

In the light of these results, one might be sanguine about the how the marriage market adjusts to secular growth in cohorts, in the demographic transition. However, our analysis shows that there is a considerable difference between the marriage squeeze due to temporary shocks, and that arising from *systematic growth* in cohort size. We find that with transferable utility, the age gap at marriage is *completely* insensitive to systematic marriage market imbalances. [4] Moreover, this is true no matter how small the relative weight of age preferences is relative to other considerations such as wealth or attractiveness. If age related preferences play a relatively minor role in men's and women's utility, a social planner could greatly reduce imbalances by mandating a reduced age gap, with only minor negative utility consequences. However, such an outcome would not arise in a marriage market equilibrium. Thus our analysis highlights that the demographic transition has important implications for the balance of power and resources between the sexes. It suggests that the cause of women's equality has been hindered by high cohort size growth, but that this may now be reversed in many countries, as cohort growth becomes negative.

Why is it that the age gap adjusts to transitory shocks in cohort size, but not to secular growth? Suppose that a shock reduces the size of the cohort indexed by date t. If the age gap at marriage is τ, this results in an excess supply of men at date $t-\tau$ (and an excess supply of women at $t+\tau$). Thus attractive men from date $t-\tau$ now become available. Women from cohorts that are adjacent to that affected by the shock (i.e. $t-1$ and $t+1$) now have more attractive options, and would prefer to match with these men rather than their "customary" match. Thus the age gap adjusts, and the magnitude of adjustment is greater the smaller the weight of age related preference relative to other considerations. We should therefore expect age gap adjustments, the more important considerations such as wealth, education, attractiveness or agreeability are to marrying individuals, as compared to considerations of age.

Consider now secular positive growth, at some rate g. The equilibrium age gap will be

[4] While our focus is on transferable utility, with non-transferable utility the equilibrium age gap may increase, aggravating the imbalance.

the preferred age gap τ^*,[5] no matter how large g is, and no matter how much of a marriage market imbalance that this causes. (Indeed, the equilibrium age gap will be τ^* even if growth were to be negative.) The basic reason is that there is no margin for profitable adjustment available. In a steady state equilibrium, there will be an excess of women in every cohort. So the unmatched women from cohort t are as attractive as unmatched women from any other cohort $\tilde{t}$. This implies that there is no incentive to choose a woman other than from ideal age gap, τ^*. Indeed, this is true no matter how weak the preferences for a specific age gap are, relative to other characteristics.

Our focus on marriage market flows differs from that of the large literature on the number of "missing women" in the population stock (see Sen (1990), Coale (1991) and Anderson and Ray (2010), for a very partial list). This is not to deny the importance of missing women overall, but rather because they are unlikely to have similar behavioral consequences.[6]

This paper is related to several strands of literature. First, there is the literature on marriage markets, following Gale and Shapley (1962) who assume non-transferable utility and Shapley and Shubik (1972) and Becker (1981), who assume transferable utility. More specifically, there is the literature on the marriage squeeze, by demographers as well as economists, including Akers (1977), Schoen (1983), Bergstrom and Lam (1989a, 1989b), Bhatt and Halli (1999), Anderson (2007), Brandt, Siow and Vogel (2008) and Gupta (2013). Second, our work is related to a large volume of work on empirical work on the sex ratio. Apart from the literature on missing women, Neelakantan and Tertilt (2008) is particularly relevant since they focus on the marriage market. Finally, there is a literature on dowries and the reason for their increase in South Asia. This includes Rao (1993), Anderson (2003, 2007a, 2007b), Ambrus, Field and Torreo (2010).

The organization of the rest of the paper is as follows. Section 2 sets out the empirical evidence on cohort growth and marriage market balance in number of developing countries in Asia and Africa. Section 3 sets out a transferable utility model of the marriage market, and analyzes its steady state. We ask if the marriage market permits an adjustment mechanism, via the endogeneity of the age gap. Section 4 considers the implications for the relative position of the two sexes, and the effects on dowries. Section 5 turns to transitory shocks and shows that adjustment in the age gap does take place, and will be greater the less important age considerations are relative to considerations of quality. Section 6 analyzes a non-transferable utility model and shows that our essential conclusions are robust. The final section concludes.

[5] More precisely, τ^* is defined as the age gap that maximizes the sum of payoffs of the two sexes.

[6] One could argue that if women are more abundant, their greater political voice may ensure that public polcy is more female friendly; however, this mechanism reinforces imbalances rather than correcting them.

2 Marriage market balance

The effective supply-demand situation in the marriage market depends upon the sex ratio at birth, and upon rate of growth of female cohorts relative to the males that they are matched with. Consider a society where cohort sizes are growing at an annual rate g. Let the age gap at marriage, between men and women, be given by τ years. The *required number of boys*, R, per 100 girls, is given by

$$R = 100(1+g)^{\tau}\frac{\Delta_G}{\Delta_B}\frac{\lambda_G}{\lambda_B},$$

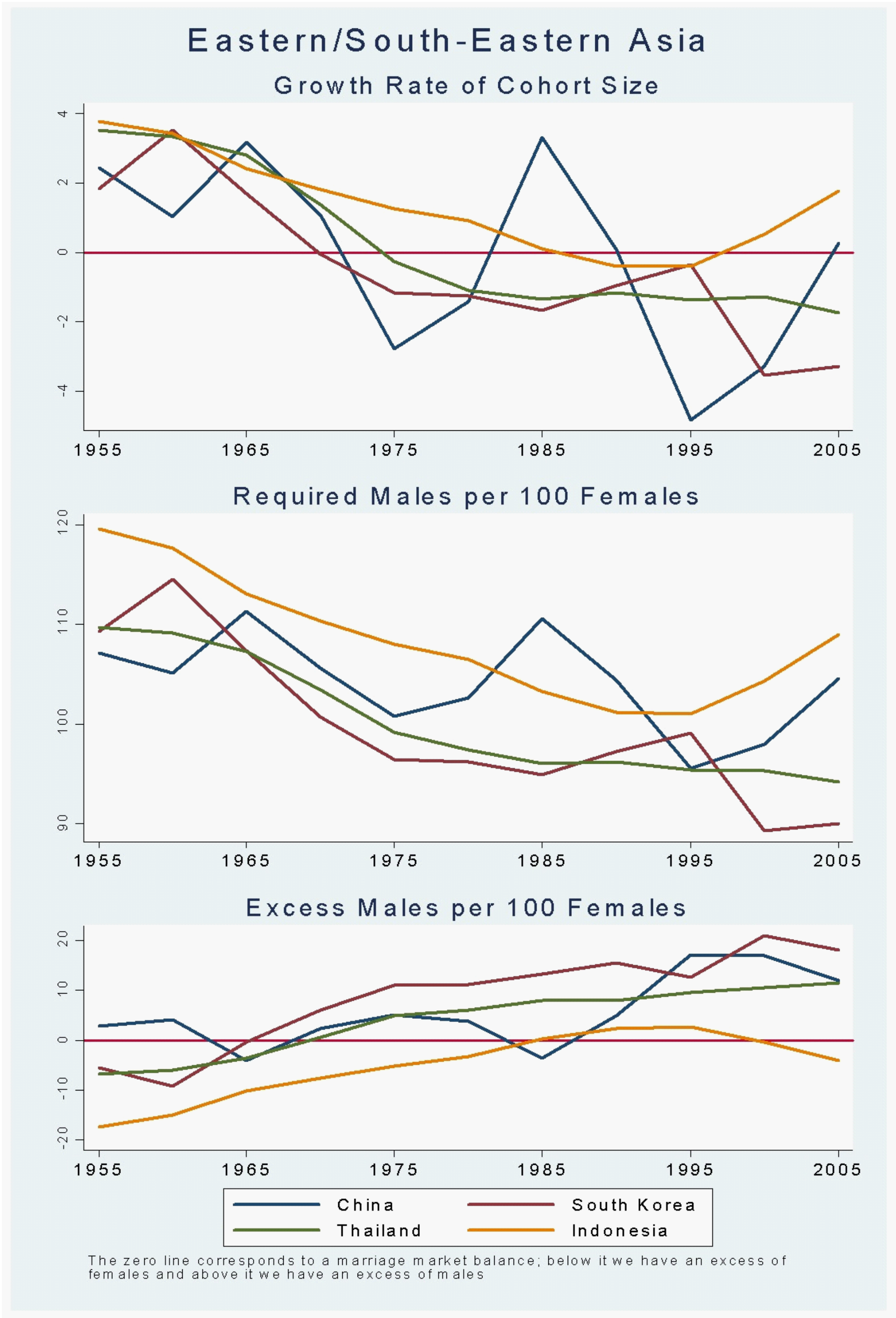
where Δ_B is the survival rate for boys, between infancy and the age of marriage, and Δ_G is that for girls. λ_G (resp. λ_B) is the proportion of girls (resp. boys) who would like to marry.[7] Thus if the marriage market is to be balanced, the actual number of boys per hundred girls, A, must be close to or equal to R. The difference $G = A - R$ measures the extent to which there is an excess of men (or missing women) on the marriage market.

We now present estimates from a range of countries in Asia and Africa on g, R, and G. Population growth is estimated using the UN's "World Population Prospects: The 2012 Revision" data on population by sex in the age group 0-4, at five yearly intervals starting 1950. Since cohort data is noisy, g for any year t is based on a regression estimate of the growth rate using observations at t, $t-5$ and $t+5$. To estimate survival rates, we use infant mortality data for each year from the same dataset, since this is the main component of mortality. Since it is problematic to use observed data on the proportions marrying in order to estimate λ_G and λ_B, we use the singles rate in the age group 50-54 from the UN's "World Marriage Data 2012". The age gap is also from this dataset, and based on the difference between the singulate mean age at marriage between men and women. The singles rate and the age gap is taken at a fixed year; 1990 or the closest year with available data.

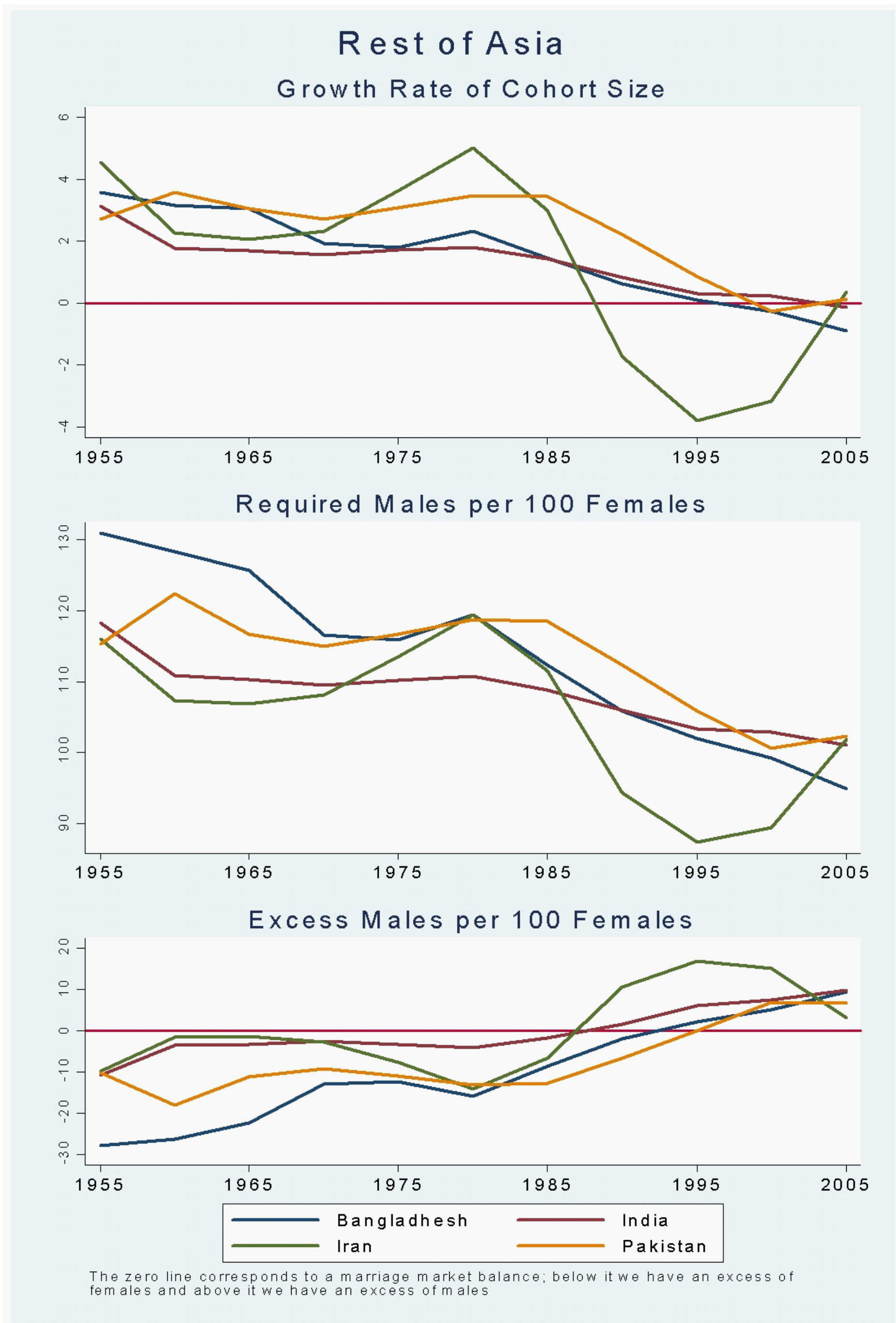
The charts in figures 1-4 present the evolution of g, R and G in four groups of countries: East/South-East Asia, South Asia (and Iran), North Africa, and sub-Saharan Africa. Within each group we present a selection of countries, mainly for reasons of space – we have constructed these measures for all countries in Asia and Africa (and selected countries in other continents), and the trends there are similar. The main features that these charts illustrate are as follows:

- Cohorts were growing rapidly, in the 1950s and 1960s, in almost all developing countries, and this resulted in a large excess supply of women, in the marriage markets of

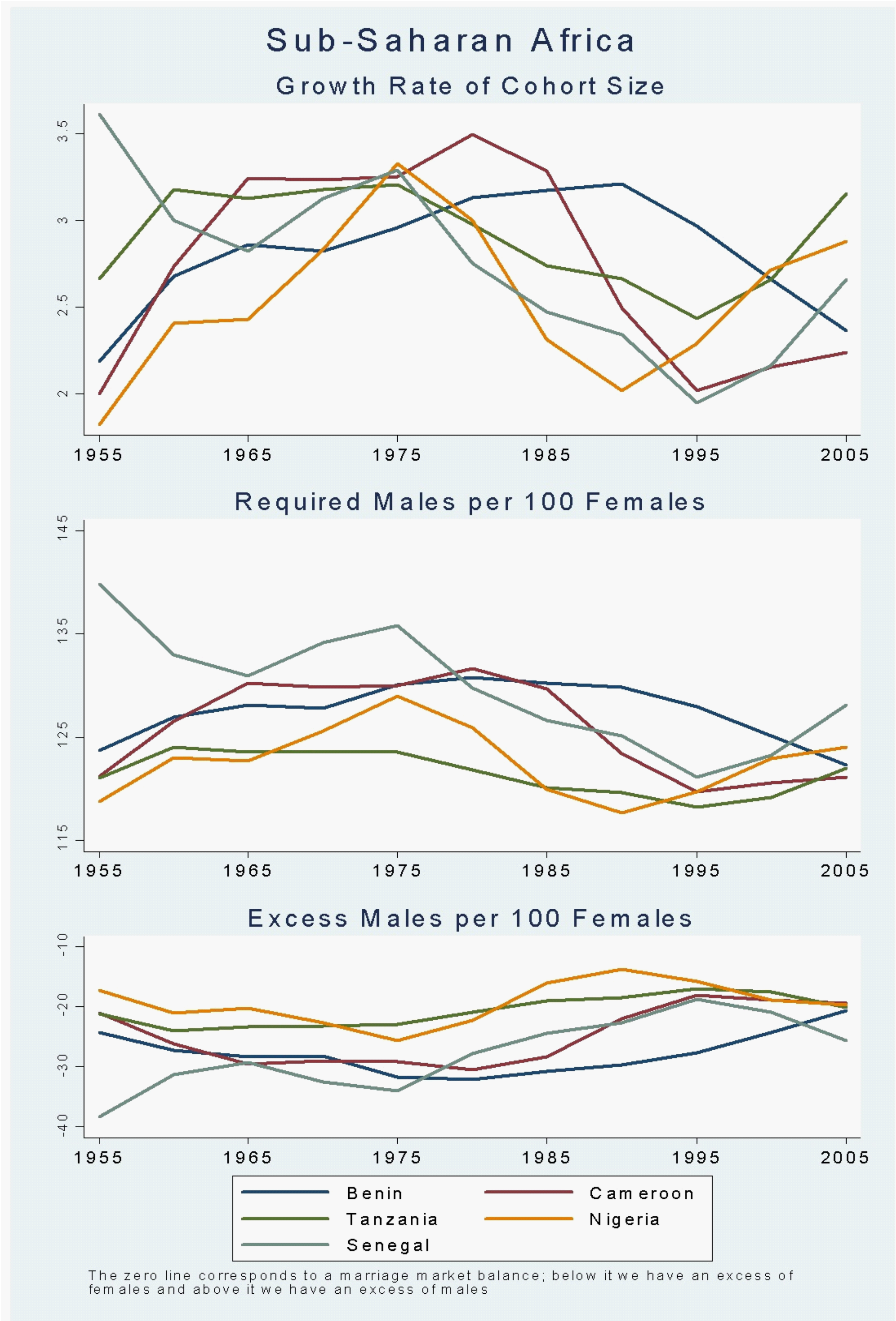
[7] This is valid if λ_G and λ_B are the proportions who *desire* marriage. However, in empirical work, demographers use the observed proportions, and we have some reservations about this, since the actual proportions of women and men that marry will reflect marriage market conditions, i.e. will be endogenous. As we shall see, our substantive conclusions are not much affected by this adjustment.

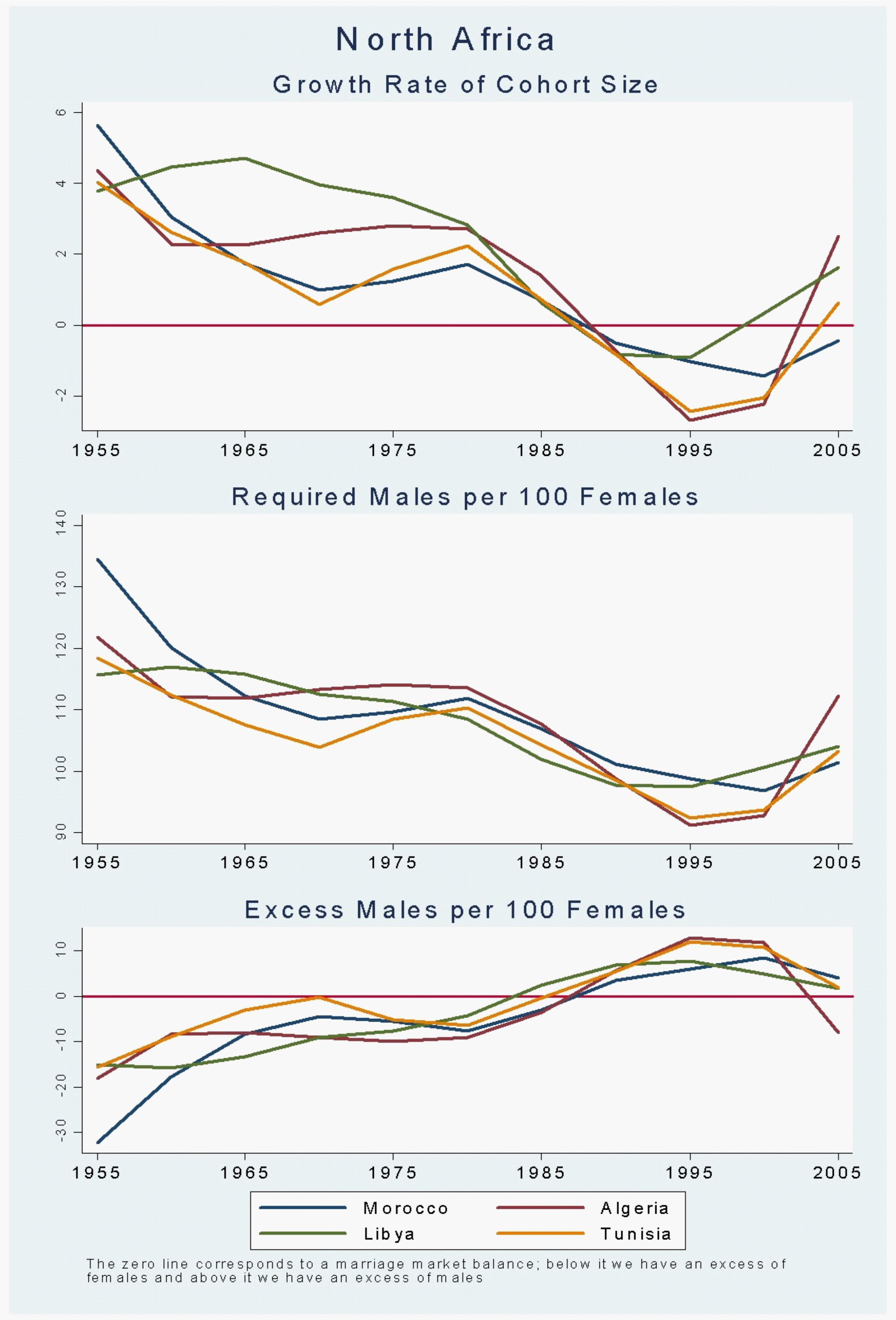


Eastern/South-Eastern Asia
Growth Rate of Cohort Size
4
2
0
-2
-4
1955
1965
1975
1985
1995
2005
Required Males per 100 Females
120
110
100
90
1955
1965
1975
1985
1995
2005
Excess Males per 100 Females
20
10
0
-10
-20
1955
1965
1975
1985
1995
2005
China
South Korea
Thailand
Indonesia
The zero line corresponds to a marriage market balance; below it we have an excess of females and above it we have an excess of males



Rest of Asia
Growth Rate of Cohort Size
Required Males per 100 Females
Excess Males per 100 Females
1955
1965
1975
1985
1995
2005
Bangladhesh
India
Iran
Pakistan
The zero line corresponds to a marriage market balance; below it we have an excess of females and above it we have an excess of males


Sub-Saharan Africa
Growth Rate of Cohort Size
3.5
3
2.5
2
1955
1965
1975
1985
1995
2005
Required Males per 100 Females
145
135
125
115
1955
1965
1975
1985
1995
2005
Excess Males per 100 Females
-10
-20
-30
-40
1955
1965
1975
1985
1995
2005
Benin
Cameroon
Tanzania
Nigeria
Senegal
The zero line corresponds to a marriage market balance; below it we have an excess of females and above it we have an excess of males


North Africa
Growth Rate of Cohort Size
Required Males per 100 Females
Excess Males per 100 Females
1955
1965
1975
1985
1995
2005
Morocco
Algeria
Libya
Tunisia
The zero line corresponds to a marriage market balance; below it we have an excess of females and above it we have an excess of males

the 1970s and 1980s.

- This phase of cohort growth ended around the 1970s (i.e. among the marriage cohorts of the 1990s) in East Asia. In other Asian countries such as India and Pakistan and Egypt, and in North Africa, phase II (of cohort growth) continued into the 1990s, and appears to be ending only now. Thus the marriage markets of today are still in a situation of excess supply of women in South Asia and North Africa.

- Most striking is the continuation of extremely large cohort growth, even today, in sub Saharan Africa, with a large excess of women.

- Cohorts are shrinking rapidly in East Asia, especially in countries such as China and South Korea. This is aggravating the problem of an excess of men. While this trend is most pronounced in East Asia, it is also apparent elsewhere, e.g. in Tunisia.

Demographers are familiar with the marriage squeeze, arising from inter-temporal variation in the size of cohorts (see e.g. Akers (1967) or Schoen (1983)). However, the focus has mainly been on the effects of transitory shocks to cohort size. In the developing country context, Bhatt and Halli (1999) and Neelakantan and Tertilt (2008) note that population growth causes marriage market imbalances. However, it is cohort growth rather than population growth that is relevant for marriage market balance, and the two variables differ quite significantly. For example, cohort growth in sub-Saharan Africa is substantially larger than population growth. Furthermore, while few countries have declining populations, cohorts are shrinking rapidly in those countries where fertility decline is pronounced. For instance, China has positive population growth, but its cohorts are shrinking at 5% per year. Thus the effective excess of men arising from cohort decline has not been appreciated.

3 A model of the equilibrium age gap

Our empirical findings raise several questions. The first relates to the endogeneity of the age gap. Our estimates in the previous section assume that the age gap at marriage, τ, is exogenous. Clearly, τ may change, for exogenous reasons. As women become more educated, their marriage age increases, while that of men does not increase by the same amount. Does the age gap at marriage adjust in order to help "clear" the marriage market? Economic "intuition" suggests that it should do so, at least on reading much of the literature on the subject. If this is the case, this could restore balance in the marriage market. A reduction in the age gap would help reduce the excess supply of women that is predicted to prevail in most parts of India. Similarly, even in China, the large excess in the actual number of boys could be reduced if the age gap fell, and even more if men began marrying older women. This point assumes greater relevance in view of the existing literature on how the age gap responds to the marriage squeeze. The pioneering work in this regard is by Bergstrom and Lam (1989a, 1989b), who study 19th century Sweden, where there were large fluctuations in the effective sex ratios across marriage cohorts, due to the age gap at marriage and the fluctuations in the size of birth cohorts. They set out an assignment model of the marriage market with transferable utility, in the style of Shapley-Shubik (1972) and Becker (1981), and conclude that marriage markets showed considerable flexibility, in the sense that age gaps at marriage adjusted relatively easily in order to clear marriage markets. More recently, Brandt, Siow and Vogel (2008) use a similar transferable utility framework to examine the effects of the large shocks to cohort size arising from the Chinese famine of 1959-61. They find that participants in the marriage market showed considerable flexibility, so that the marriage rates of the cohorts who were matched with the famine affected cohorts were damaged, but not to the extent that one might imagine.

A positive age gap seems a very pervasive phenomenon – data from the United Nations (1990), from over 90 countries, show that the difference between the ages at first marriage for men and women is positive in every single country, for every time period. This suggests that both men and women prefer a positive age gap,[8] and such preferences may have an evolutionary foundation. Kaplan and Robson (2003) present evidence from hunter-gatherer societies on age-productivity profiles. The productivity of women in gathering is relatively constant over time, while the productivity of men in hunting rises sharply from a low initial base, so that between the ages of 25 and 50, men produce a large surplus relative to their consumption requirements. In addition, women reach peak fertility relatively quickly, and their fertility declines more rapidly than that of men. In this paper, we shall assume that the pervasiveness of a positive age gap is grounded in preferences, of men as well as women. This may have evolutionary foundations. With development, as women become more educated and take on paid employment, they will prefer to delay marriage, while the preferred age of marriage for men may not rise as much. Thus the preferred age gap may fall, for exogenous reasons. An alternative approach is set out by Bergstrom and Bagnoli (1990) who attribute the age gap to the differential roles of women and men in marriage, with the suitability of women for their role (the production of offspring) being revealed earlier, while that of men in their role (bread-winning) being revealed only later. We have also briefly examined how

[8] If men preferred a positive age gap while women preferred a negative age gap, then one would expect a negative age gap to emerge in marriage markets where women are in short supply – see section 3.2.

the age gap in the Bergstrom-Bagnoli model adjusts to imbalances, and find results that are broadly consistent with those reported here.

3.1 The model: steady state analysis

Let time be discrete, and index it by the integers, positive and negative.[9] We assume a continuum population, that grows at a constant rate g per year. At each date, the relative measure of women to men and women equals $\bar{r}$, i.e. $\bar{r}$ is the sex ratio at birth or within each cohort. There are two dimensions to an individual's marriage market characteristics and preferences. Age constitutes a "horizontal dimension of differentiation, and "quality" is the vertical dimension. We assume that preferences are stationary, i.e. they do not depend upon the time index of the individuals.

In a transferable utility environment, what matters is the total payoff (or marital surplus) that can be generated by a couple, denoted by Q. This depends upon the qualities of the two individuals, and on the age gap between them. Assume that the quality of a man in any cohort, ε, is distributed with a continuous, strictly increasing cumulative distribution function $F(.)$ on $[\varepsilon_{\min}, \varepsilon_{\max}]$.Similarly, the quality of a woman, η, is distributed with with a continuous, strictly increasing cumulative distribution function $G(.)$ on $[\eta_{\min}, \eta_{\max}]$. Let τ denote the age gap, i.e. the difference between the man's age and the woman's age. Thus Q depends upon $(\varepsilon, \eta, \tau)$. We assume that it is continuous and strictly increasing in the two quality dimensions (ε and η). We also assume that for any (ε, η), Q reaches a unique maximum at $\tau^* > 0$. That is, match surplus is maximized at a positive age gap. [10] Note that this assumption is well compatible with men and women having different preferences over τ, with differing ideal points τ_B and τ_G, with τ^* lying intermediate between these points. In a transferable utility framework, what matters is total match surplus, Q, since this can be divided between the two parties in any way that they choose. The payoffs from being single are $\bar{u}$ for any man and $\bar{v}$ for any woman. We may, without loss of generality, assume that these reservation values do not depend on type of the individual.[11] We state our main assumptions as follows:

Assumption 1: Q is continuous and strictly increasing in ε and η and is maximal at τ^*. Further, $Q(\varepsilon_{\min}, \eta_{\min}, \tau^*) < \bar{u} + \bar{v} < Q(\varepsilon_{\max}, \eta_{\max}, \tau^*)$.

The second part of the assumption ensures that there are both matched and unmatched individuals in equilibrium, and also ensures that equilibrium payoffs are unique.

[9] Since we are focusing on steady states, we assume that time extends indefinitely, backwards and forwards, thus avoiding any initial date effects that would preclude existence of steady state.

[10] Our formal results would not differ if τ^* was strictly negative.

[11] If reservation values the match pair depend upon their respective types, we can re-define the match payoff as Q minus the sum of reservation values, and the same analysis applies.

Let M be the set of men and let W be the set of women. Each element of M has a pair of characteristics, $(\varepsilon, \dot{t})$, i.e. has a quality type and a cohort date. A matching is a function $\phi : \mathsf{M} \to \mathsf{W} \cup \{\emptyset\}$ that satisfies the following properties. First, if $w \in \mathsf{W} = \phi(m)$, then w is not the image of any other $m' \in \mathsf{M}$ under ϕ, i.e. any woman can be matched only to a single man. Second, if M' is any finite Lebesgue measure subset of M, such that $\phi(\mathsf{M}') \subset \mathsf{W}$, the Lebesgue measure of M' equals that of the set $\phi(\mathsf{M}')$. That is, any matching must be measure preserving.

Associated with any matching ϕ is a payoff allocation, $(u : \mathsf{M} \to \mathbb{R}, v : \mathsf{W} \to \mathbb{R})$. The payoff allocation must be feasible, i.e. for any matched pair (m, w),

$$u(m) + v(w) = Q(\varepsilon(m), \eta(w), \tau(m, w)).$$

For any unmatched individual, the payoff allocation equals his/her payoff from being single. An allocation consists of a matching and a payoff allocation, and is denoted by $(\phi, u(.), v(.))$.

We now turn to equilibrium allocations. We focus on *stable* allocations (or, more loosely, stable matchings), as defined, for a transferable utility environment by Shapley and Shubik (1972). Stability implies that $u(m) \geq \bar{u}$ and $v(w) \geq \bar{v}$, i.e. any individual gets at least the payoff from being single. Secondly, for any (m, w) who are not matched to each other, $u(m) + v(w) \geq Q(\varepsilon(m), \eta(w), \tau(m, w))$. That is, the payoff allocations of this pair must be weakly greater than the total match value that they could generate by matching together, since otherwise, this pair would have a profitable pair-wise deviation from the matching.

Our final requirement is that we shall restrict attention to steady states, where the matching pattern is stationary over time, so that a man of type ε and at any date t is matched with a woman of type $\phi(\varepsilon)$ at date $t + \tau(\varepsilon)$. That is, the matching $\phi(.)$ is not indexed by t. The steady state requirement, of course, applies only to equilibrium matches – deviating couples could have an arbitrary age gap.

The simplest steady states are monomorphic steady states, where every couple that matches has the same age gap, τ, so that $\tau(\varepsilon)$ does not depend upon ε. In contrast, in a polymorphic steady state, the matching pattern is stationary, but where the age gap can depend upon the man's type.

In a monomorphic steady state with age gap τ, men born at date t will be matched with women born at $t + \tau$, and the ratio of the latter to the former will equal $r = \bar{r}(1 + g)^{\tau}$. Since the matching must be measure preserving, if $r > 1$, a there will be more unmatched females than males in every cohort, while if $r < 1$, the reverse is the case.

We are now in a position to state the main result of this section, but before doing this, let us put it in context. In finite economies with transferable utility, Shapley and Shubik (1972) show that finding a stable matching is equivalent to finding an assignment of men to women that maximizes total surplus. That is, a stable matching is the solution to a linear-programming assignment problem. Gretsky, Ostroy and Zame (1992) extent this result to continuum economies with a finite measure of agents. Our society has a countable infinity of dates, so that the total measure is not finite. In consequence, total surplus is not well defined, since the infinite sum of payoffs associated with an allocation is not well defined, the series being non-convergent. This precludes using the existing results, and our proof of existence and uniqueness must be done directly. By using the results of Gretsky et. al., we are able to show there exists a stable allocation in a static problem, and we use this to show the existence of stable allocation in the dynamic context.

Theorem 1 *Under assumption 1, there exists a steady state stable matching and a unique steady state payoff allocation,* (u^*, v^*)*. In a steady state stable match, the age gap equals* τ^**, independent of* g*.*

Proof. Consider the static matching problem where men from cohort t are matched with women from cohort $t+\tau^*$, so that the measure of women relative to the measure of men equals $r^* = \bar{r}(1+g)^{\tau^*}$. By Gretsky, Ostroy and Zame (1991), there exists a stable match ϕ in this matching problem with payoffs $u^*(\varepsilon)$ for each type of man and $v^*(\eta)$ for each type of woman. Now consider the dynamic matching problem, and construct $\tilde{\phi}$ from ϕ as follows. For any man of type ε and for every t, let this man be matched with a woman of type $\phi(\varepsilon)$ of index $t+\tau^*$ (if type ε is unmatched under ϕ, he is also unmatched under $\tilde{\phi}$). Let the payoff allocation be $u^*(\varepsilon)$ and $v^*(\eta)$ be the payoff allocation, i.e. it coincides with the payoffs in the static matching problem. We show that this steady state allocation is stable. First, consider any type pair (ε, η) with age gap τ^*. Since the static matching ϕ is stable, $Q(\varepsilon, \eta, \tau^*) \leq u^*(\varepsilon) + v^*(\eta)$, implying that the matching $\tilde{\phi}$ cannot be blocked by this pair. Now consider any type pair (ε, η) with age gap $\tau \neq \tau^*$. Thus $Q(\varepsilon, \eta, \tau) < Q(\varepsilon, \eta, \tau^*) \leq u^*(\varepsilon)+v^*(\eta)$, so that the matching $\tilde{\phi}$ cannot be blocked by this pair either. This verifies that $\tilde{\phi}$ and the allocation $u^*(.)$,$v^*(.)$ is stable.

To show that there is no other stable steady state, suppose that there is a polymorphic steady state where some type ε is matched to type η with an age gap $\tau \neq \tau^*$. Let them earn payoffs $u(\varepsilon)$ and $v(\eta)$ respectively, where $u(\varepsilon)+v(\eta) = Q(\varepsilon, \eta, \tau) < Q(\varepsilon, \eta, \tau^*)$. Then the man of quality ε in an arbitrary cohort can propose a match to the woman of the same quality η, but with age gap τ^*. Since the total payoff of this match is $Q(\varepsilon, \eta, \tau^*) > u(\varepsilon) + v(\eta)$, the candidate matching is not be stable. ∎

It is noteworthy that this invariance result does not depend upon how important τ is relative to the quality dimension (ε, η) in the overall match payoff function $Q(.)$. For instance, considerations of age could be arbitrarily unimportant, so that deviations from τ^* reduce Q very little. However, a reduction in τ below τ^* could significantly increase the number of matches, when g differs from zero, by making the sex ratio more balanced.

The main content of theorem 1 is that the age gap *does not adjust* to changes in g or $\bar{r}$ that result in a marriage market imbalance. That is, the simple intuition of supply and demand does not operate in the context of the marriage market, due to the indivisibility in the assignment problem, whereby one man can be assigned to at most one woman, and vice versa. [12] Under transferable utility – i.e. precisely the assumptions made by Bergstrom-Lam and Brandt et al. – the age gap does not adjust in response to changes either in the sex ratio at birth or changes in the rate of growth of cohort size. Indeed, this result applies even if preferences regarding the age gap have relatively little weight in the utility functions of men and women – note that our assumptions allow for the possibility that the total match surplus declines very little as the age gap changes from τ^*. Things are quite different if we consider a *transitory shock* to cohort size, as we see in section 5. For example, take the case where $\bar{r} = 1$ and $g = 0$ and $\tau_B = \tau_G = \tilde{\tau} > 0$, so that the steady state marriage market sex ratio is balanced. Suppose that there is positive shock to the birth cohort at date t. In this case, there is an excess supply of women born at date t (whose ideal match, of men born at $t - \tilde{\tau}$, is smaller) and an excess supply of women born at date t. This will reduce the price of men and women born at date t, and raise the price of those born in scarce cohorts. Since some types of women are scarce, and some types of men are scarce, these price changes induce adjustment in the age gap of the affected cohorts and those nearby, and magnitude of these adjustments will be sensitive to the weight that age sensitive preferences have. In contrast, a long run increase in the marriage market sex ratio, say due to a rise in g, has the effect of making all women more plentiful. Thus there are no relative price effects across cohorts that induce an adjustment in the age gap.[13]

Our model predicts a single age gap, while in reality there is a distribution of age gaps. This can be generated, by introducing shocks to preferences, as in Choo and Siow (2006). Alternatively, one may explicitly model heterogeneous preferences, as we have done in a previous version of this paper. It suffices for our purpose to note that introducing heterogeneity does not affect our main result, that the distribution of age gaps does not depend upon cohort growth.

[12] It is possible that if there is an excess supply of women (men), they may be more willing to accept polygamy (polyandry). This would be the case if the total payoff of a *menage a trois* is greater than the payoff a couple plus a single. We leave an exploration of this issue to future work.

[13] Sautmann (2012) argues, using a search model, that the marriage squeeze is responsible for a declining age gap in India. Her model is complex, and since she cannot solve it analytically, she uses numerical simulations. It is therefore unclear whether this is a robust result.

3.1.1 Some examples

An explicit example, where we have positive assortative matching, is illustrative. Let Q be strictly supermodular in (ε, η). Recall that $r^* = \bar{r}(1+g)^{\tau^*}$ be the consequent sex ratio in the marriage market. Supermodularity implies that matching is assortative in the quality dimension. Let $\tilde{\varepsilon}$ and $\tilde{\eta}$ denote the lowest matched types of men and women respectively, where $\phi(\tilde{\varepsilon}) = \tilde{\eta}$. $\tilde{\varepsilon}$ and $\tilde{\eta}$ are the unique solutions to the pair of equations:

$$1 - F(\tilde{\varepsilon}) = r^*[1 - G(\tilde{\eta})]. \tag{1}$$

$$Q(\tilde{\varepsilon}, \tilde{\eta}, \tau^*) = \bar{u} + \bar{v}. \tag{2}$$

Since the matching is assortative, $\phi(\varepsilon) \geq \tilde{\eta}$ is defined, for $\varepsilon \geq \tilde{\varepsilon}$ by

$$1 - F(\varepsilon) = r^*[1 - G(\phi(\varepsilon))]. \tag{3}$$

Having defined payoffs for the marginal types in the market, we now turn to the rest. Let $v^*(\eta)$ denote the payoff of a woman of type η. Let $\phi(\varepsilon)$ denote the match of a man of type ε. Stability implies that $\phi(\varepsilon)$ is the optimal choice for a man of type ε, i.e.

$$\arg\max_{\eta} \left[Q(\varepsilon, \eta, \tau^*) - v^*(\eta)\right] = \phi(\varepsilon).$$

The first order condition yields the differential equation

$$Q_\eta(\varepsilon, \phi(\varepsilon), \tau^*) = v^{*\prime}(\phi(\varepsilon)).$$

The solution to this differential equation, in conjunction with the boundary condition, that $v^*(\tilde{\eta}) = \bar{v}$, gives us the payoffs all types of matched women. Thus, the payoff of any woman of type $\eta \geq \tilde{\eta}$ is given by

$$v^*(\eta) = \bar{v} + \int_{\tilde{\eta}}^{\eta} Q_y(\phi^{-1}(y), y, \tau^*)dy.$$

The payoff to a matched man equals the residual, $Q(\varepsilon, \phi(\varepsilon), \tau^*) - v^*(\phi(\varepsilon))$. The payoffs to unmatched men, of type $\varepsilon < \tilde{\varepsilon}$, equals $\bar{u}$. The payoff to unmatched women, of type less than $\tilde{\eta}$, equals $\bar{v}$. This defines a payoff allocation for every type of man and woman, $(u^*(\varepsilon), v^*(\varepsilon))$. Note that this payoff allocation depends upon the sex ratio r^*, which in turn depends upon the age gap.

A second example is one $Q(\varepsilon, \eta, \tau^*)$ is sub-modular in (ε, η), in which case we have negative assortative matching. Finally, if Q is additive in ε and η,e.g. $Q(\varepsilon, \eta, \tau^*) = \varepsilon + \eta$, then the matching pattern is indeterminate, with all types $\varepsilon \geq \bar{u}$ and $\eta \geq \bar{\eta}$ being matched. We will examine the additive case in greater detail when we discuss dowries.

For general quality functions $Q(.)$, it is well known that the static matching ϕ is difficult to characterize, being NP-hard. Nonetheless, theorem 1 allows us to characterize the equilibrium age gap, in the dynamic context even when we are unable to characterize ϕ.

3.1.2 Welfare considerations

In marriage markets with finitely many agents and transferable utility, the Shapley-Shubik results imply that the first and second welfare theorems hold. With transferable utility, welfare equals the sum of payoffs across all agents, and a stable matching maximizes this, implying the first welfare theorem. Furthermore, if lump sum transfers across agents are feasible, they can be used to redistribute payoffs, without any consequences for the matching pattern. Thus the second welfare theorem also holds.

In the present context, the sum of payoffs across all agents is not well defined since the infinite series is necessarily non-convergent. If $g \geq 0$, so that cohort sizes are growing or constant, then the infinite series is clearly non-convergent. However, this is true even if $g < 0$, since the infinite sum going backwards in time is non-convergent. So we must adopt alternative welfare criterion. One possibility is to consider average per-capita payoffs. However, even here there is some ambiguity. Consider a steady state where the age gap is τ, and where the associated marriage market sex ratio equals $r = \bar{r}(1+g)^{\tau}$. Assume that payoffs are weakly supermodular, so that efficiency requires that matching is assortative. Thus the lowest types on the long side of the market are unmatched, and for a man of type ε who is matched, $\phi(\varepsilon)$, satisfies

$$[1 - F(\varepsilon] = r[1 - G(\phi(\varepsilon))].$$

The total surplus associated with matches between men at date t and women at date $t+\tau$ equals

$$y(\tau) \equiv \int_{\underline{\varepsilon}}^{\varepsilon_{\max}} Q(q(\varepsilon, \phi(\varepsilon)), S(\tau))dF(\varepsilon).$$

Now consider the welfare per man at date t. The surplus generated per man equals $y(\tau)$ if all men are matched, i.e. if $r \geq 1$. However, if $r < 1$, then the surplus per man equals $ry(\tau)$. That is,

$$W_m(\tau) = \min\left\{y(\tau), ry(\tau)\right\}.$$

Consider instead the surplus the surplus generated per woman at date t. This is given by

$$W_f(\tau) = \min\left\{y(\tau), \frac{1}{r}y(\tau)\right\}.$$

A third possible welfare criterion is the surplus generated per person in each cohort. Since the proportion of women to men within cohort is $\frac{1}{1+\bar{r}}$, this is equal to $\frac{1}{1+\bar{r}}W_m(\tau) + \frac{\bar{r}}{1+\bar{r}}W_f(\tau)$, i.e. it is a convex combination of the previous two welfare measures.

The steady state stable age gap τ^* does not, in general, maximize any of these welfare criteria. To examine this issue further, let us consider the expression $Q(q(\varepsilon, \phi(\varepsilon)), S(\tau))$ at any realization of ε. Since $\phi(\varepsilon)$ is increasing in r (i.e. a man of a given quality gets a better match if the sex ratio increases), this implies that $q(\varepsilon, \phi(\varepsilon))$ is increasing in r. The second argument of Q is $S(\tau)$ and this is single peaked around τ^*. This establishes that at τ^*, a change in τ reduces $W_m(\tau)$ if this change in τ reduces $\dot{r}$; however, this change in τ may increase $W_m(\tau)$ if the consequence is to increase r, and thereby $q(\varepsilon, \phi(\varepsilon))$ sufficiently. This argument is summarized in the following proposition.

Proposition 2 *The stable age gap τ^* does not, in general, maximize $W_m(\tau)$ or $W_f(\tau)$. If $r(\tau^*) = \bar{r}(1+g)^{\tau^*} < 1$ and age preferences are sufficiently weak, then $W_m(\tau)$ is maximized at a value of τ that increases r relative to $r(\tau^*)$. If $r(\tau^*) = \bar{r}(1+g)^{\tau^*} > 1$ and age preferences are sufficiently weak, then $W_f(\tau)$ is maximized at a value of τ that reduces r relative to $r(\tau^*)$.*

4 Distributional effects

What are distributional effects of changes in the effective sex ratio in the marriage market, whether due to the marriage squeeze or due to sex selective abortions? Our analysis has shown that equilibrium payoffs of men and women in the steady state coincide with equilibrium payoffs in a static model where the sex ratio is $r^* = \bar{r}(1+g)^{\tau^*}$. We may therefore consider the effects of a change in r, in the static model. Clearly, if r^* increases, men benefit and women lose out, due to a worsened competitive position. This occurs regardless of whether there is transferable utility or non-transferable utility, since more men will be able to marry, while less women are able to. However, if utility is transferable, there will be *distributional* effects, even on women who are able to find partners. We now show that the *magnitude* of distributional effects depends upon the nature of the marriage market.

Specifically, the distributional effects of sex ratio imbalances will be large in Asian societies, while they will be more muted in North European societies.

To elucidate the reasons for this difference, one must consider the difference between marriage institutions across cultures. In his seminal work, Hajnal (1965) pointed out the difference between marriage patterns in Northern Europe and Southern or Eastern European society. As Hajnal (1982) observes, these differences are accentuated all the more when one compares Northern Europe with Asia – i.e. Southern or Eastern Europe lie somewhere in between the Asian and Northern European marriage pattern. The salient features are as follows:

- A high age at marriage for both men and women (NE), as compared to earlier marriage.
- A small age gap at marriage (NE).
- A large fraction of the population who never married (NE), with a never married rate of about % (NE), as compared to a 98 or 99% marriage rate in Asia.

Clearly, a large age gap at marriage magnifies the marriage squeeze, both due to temporary shocks and due to secular growth. For example, a 3.5% rate of growth translates into a 19% excess supply of women if the age gap is five years, but only a 7% excess supply if the age gap is 2 years – such are the dramatic effects of compound growth.

More subtle is the effect of the last factor – the distributional effects of the marriage squeeze will be large in societies where marriage is near universal, but much smaller in North European type societies, where marriage rates are lower.

Let us now consider the effects of a change in r^*, that may arise either due to a worsening of the sex ratio at birth, or due to a change in the age gap. An increase in r^* raises the relative supply of women. From the conditions for the marginal type (1) and (2), one may verify that this increases $\tilde{\eta}$ and reduces $\tilde{\varepsilon}$. That is, the the singles rate for men declines and that of women increases. Turning to payoffs, consider the payoffs of a man of a given type, ε. This can be written as

$$u^*(\varepsilon, r) = \bar{u} + \int_{\tilde{\varepsilon}(r)}^{\varepsilon} u'(x)dx.$$

Recall that the derivative, $u'(\varepsilon)$, is given by

$$u'(\varepsilon) = q_{\varepsilon}(\varepsilon, \phi(\varepsilon, r)).$$

Thus the payoff can be written as

$$u^*(\varepsilon, r) = \bar{u} + \int_{\tilde{\varepsilon}(r)}^{\varepsilon} q_x(x, \phi(x, r))dx.$$

The derivative of this payoff with respect to a change in the equilibrium sex ratio, r, is given by

$$\frac{\partial u(\varepsilon)}{\partial r} = \int_{\tilde{\varepsilon}(r)}^{\varepsilon} q_{xy}(x, \phi(x, r))\frac{\partial \phi(x, r)}{\partial r}dx - q_x(x, \phi(x))|_{x=\tilde{\varepsilon}(r)} \frac{\partial \tilde{\varepsilon}}{\partial r}.$$

Suppose now that the payoff function is strictly supermodular. Then $q_{xy} > 0$, and $\frac{\partial \phi(x,r)}{\partial r} > 0$, i.e. an increase in the supply of women improves the match quality of any type of man. Thus the first term is strictly positive. Furthermore, $\frac{\partial \tilde{\varepsilon}}{\partial r} < 0$, i.e. the marginal type of man worsens. Since q_x is positive, the second term is also positive, and the payoff of any type of man increases. The same argument implies that the payoff of any type of woman worsens.

If the payoff if weakly supermodular, so that $q_{xy} = 0$, then the first term is zero, but the second effect persists, so that an increase in the sex ratio worsens the position of women and increases that of men. In this case, we may write the payoff $q(x, y) = x + y$, so that

$$u^*(\varepsilon, r) = \bar{u} + [\varepsilon - \tilde{\varepsilon}(r)].$$

This verifies that the payoff of men increases at a rate that depends upon how quickly $\tilde{\varepsilon}$ declines as r increases. Thus in the case where the payoff function is additive, distributional effects will be larger, the greater the change in the marginal type induced by the sex ratio. From the conditions (1) and (2), the derivative of the marginal type of woman, as a function of the sex ratio, is given by

$$\frac{d\tilde{\eta}}{dr} = -\frac{1 - G(\tilde{\eta})}{f(\tilde{\varepsilon})\frac{q_\eta}{q_\varepsilon} + rg(\tilde{\eta})} = -\frac{1 - G(\tilde{\eta})}{f(\tilde{\varepsilon}) + rg(\tilde{\eta})}.$$

Thus the distributional effects are larger if the densities associated with the marginal types, $f(\tilde{\varepsilon})$ and $g(\tilde{\eta})$, are small. Suppose that ε and η are given by single peaked distributions, where f and g are strictly increasing up to a point and then strictly decreasing. Consider two different societies. First, an Asian one where $\bar{u}$ and $\bar{v}$ are low, so that the equilibrium marriage rate is high, over 95%. Second, a North European one where $\bar{u}$ and $\bar{v}$ are high, so that the equilibrium marriage rate is low, say below 90%. The numerator will be smaller in the latter, and the denominator will be larger. We summarize our results in the following proposition.

Proposition 3 *An increase in r^*, induced by a change in the sex ratio or in cohort size* *growth, increases the singles rate for women and reduces that for women. The payoff for a non-marginal type of man strictly increases, and that for a non-marginal woman falls. If payoffs are additive, the magnitude of distributional effects is larger in societies where the density of marginal types is sparse.*

4.1 The effect on dowries

Our analysis has focused on the payoffs to men and women, as a function of the sex ratio. This is separate from the effects on dowries (or bride-prices), which are monetary transfers at the time of the marriage. Becker has argued that dowries arise due to the inflexibility in the division of the marital surplus within the marriage. Under this interpretation, the total marital output, $Q(\varepsilon,\eta)$, is divided into *premuneration values* $U(\varepsilon,\eta)$ and $V(\varepsilon,\eta)$, for the man and woman respectively (see Mailath, Postelwaite and Samuelson (2013), who introduce this notion). These will, in general, differ from the payoffs in the stable allocations $u^*(\varepsilon)$ and $v^*(\eta)$, and a dowry or brideprice is paid to reconcile the two. Our analysis speaks to the equilibrium payoffs, not dowries per se. If the equilibrium premuneration value of given type of woman, does not respond as quickly to an increase in r^* as the equilibrium payoff $v^*(\eta,r^*)$, then the dowry paid must rise. However, it is also possible that premuneration values for some types rise rapidly enough that dowries may even fall.[14] The reason is, as r^* increases, each man is matched with a woman of higher quality, since $\phi(\varepsilon)$ increases,with r. This may induce a large increase is premuneration value for the man.

Let a man's quality type be $\boldsymbol{\varepsilon}=(\varepsilon_1,\varepsilon_2)$, and similarly the woman's type be $\boldsymbol{\eta}=(\eta_1,\eta_2)$. The first dimension, ε_1 (or η_1) measures the attractiveness of the individual to the match partner, or his/her pizzaz. The second dimension, ε_2 (or η_2) measures the intensity of the individual's desire for marriage. We shall assume that total match quality q is given by

$$q=\lambda(\varepsilon_1+\eta_1)+((1-\lambda)(\varepsilon_2+\eta_2)+\rho\varepsilon_1\eta_1,$$

where $\rho>0$ so that there is supermodularity in the attractiveness dimension. Indeed, we shall focus on the limit case as $\rho\to 0$, so that payoffs are additive in the limit. Assume that pizzaz and desire are independently and uniformly distributed on the interval $[0,1]$. The payoffs from being single are $\bar{u}=\bar{v}=\frac{1}{4}$.

Define $\varepsilon=\lambda\varepsilon_1+(1-\lambda)\varepsilon_2$, similarly $\eta=\lambda\eta_1+(1-\lambda)\eta_2$. Clearly, selection into matching only depends upon the one dimensional variables ε and η. Similarly, the equilibrium payoffs

[14] There is a controversy on whether an increase in dowries can be attributable to the marriage squeeze – see Anderson (2003, 2007a, 2007b).

only depend upon ε and η, and may therefore be written as $u^*(\varepsilon)$ and $v^*(\eta)$. However, since matching is assortative based on attractiveness, as long as $\rho > 0$, we shall assume that this is the case in the limit as well.

Consider first the case where the sex ratio is balanced, i.e. when $r = 1$. The equations for the marginal types in the uniform case for arbitrary r are

$$\tilde{\varepsilon}(r) + \tilde{\eta}(r) = \bar{u} + \bar{v}, \tag{4}$$

$$1 - \tilde{\varepsilon}(r) = r\left[1 - \tilde{\eta}(r)\right]. \tag{5}$$

Evaluating at $r = 1$, we find that $\tilde{\varepsilon}(1) = \frac{1}{4}$ and $\tilde{\eta}(1) = \frac{1}{4}$. Thus types with $\varepsilon \geq \frac{1}{4}$ and $\eta \geq \frac{1}{4}$ find a partner. Thus the equilibrium payoffs are given by

$$u^*(\varepsilon; r = 1) = \frac{1}{4} + (\varepsilon - \frac{1}{4}) = \varepsilon = \lambda\varepsilon_1 + (1 - \lambda)\varepsilon_2.$$

$$v^*(\eta; r = 1) = \frac{1}{4} + (\eta - \frac{1}{4}) = \eta = \lambda\eta_1 + (1 - \lambda)\eta_2.$$

Let us now turn to premuneration values. The *equilibrium* premuneration value, $\hat{U}(\varepsilon, r)$, is defined by

$$\hat{U}(\varepsilon; r) \equiv U(\varepsilon, \phi(\varepsilon; r)).$$

Thus, when $r = 1$,

$$\hat{U}(\varepsilon; r = 1) = (1 - \lambda)\varepsilon_2 + \lambda\phi(\varepsilon_1; r = 1).$$

Since $\phi(\varepsilon_1) = \varepsilon_1$ when $r = 1$,

$$\hat{U}(\varepsilon; r = 1) = (1 - \lambda)\varepsilon_2 + \lambda\varepsilon_1 = \varepsilon.$$

Similarly, $\hat{V}(\eta; r = 1) = \eta$. Thus premuneration values equal the equilibrium payoffs for every type, implying that dowries are zero for every individual when $r = 1$.

Now consider a higher ratio of women to men, e.g. the case when $r = 2$. Solving equations 4 and 5 we see that $\tilde{\varepsilon}(2) = 0$ and $\tilde{\eta}(2) = 0.5$. Thus all men are matched, and women with $\eta \geq \frac{1}{2}$ will be matched. Since the lowest type of man must get his outside option, the payoffs of the matched men and women satisfy

$$u^*(\varepsilon, r=2) = \frac{1}{4} + \varepsilon.$$

$$v^*(\eta, r=2) = -\frac{1}{4} + \eta.$$

Thus the payoff of each type of man increases by $\frac{1}{4}$,while of every type of woman falls by $\frac{1}{4}$.

While the equilibrium payoffs do not depend upon the exact value of λ, the premuneration values do. Consider the case where $\lambda = 0$ so that individuals type only differ in their desire for marriage.Thus

$$\hat{U}(\boldsymbol{\varepsilon}, r) = \varepsilon,$$

for every man. Since the dowry $d(\varepsilon)$ equals the difference between $u^*(\varepsilon)$ and $\hat{U}(\varepsilon)$, when $r = 2$,

$$d(\varepsilon) = (\frac{1}{4} + \varepsilon) - \varepsilon = \frac{1}{4}.$$

Thus the dowry does not depend upon type, and equals 0.25 in every match. The effect of an increase in the effective sex ratio is to cause dowries to rise uniformly. Thus the marriage squeeze provides an explanation for the increase in dowries in India, and the spread of the phenomenon of dowries to areas where there were not previously prevalent, such as Bangladesh and some communities in India. This is an argument that has been made before, e.g. Rao (1993). [15]

However, the effect on dowries can be more subtle when both attractiveness and the desire for marriage are important in determining match value. Consider the case where $\lambda = 0.5$, so that both dimensions matter equally. When $r = 2$, all men will be matched, and women are matched if $\eta_1 + \eta_2 \geq 1$. Thus the marginal distribution of matched women is increasing in attractiveness, and is given by

$$\tilde{g}(\eta_1) = 1 - \eta_1.$$

Thus the matching, $\phi(\varepsilon_1)$ is defined by

$$1 - \varepsilon_1 = 2\left[\phi(\varepsilon_1) - \frac{(\phi(\varepsilon_1))^2}{2}\right],$$

which yields

[15] Anderson (2007a) argues that the marriage squeeze cannot explain the rise in dowries. However, she considers a transitory shock to cohort size, rather than a secular trend.

$$\phi(\varepsilon_1) = 1 - \sqrt{1-\varepsilon}.$$

Thus partner quality is an increasing and concave function of the man's attractiveness. The quality of matched women spans the entire range from 0 to 1. Match quality rises sharply initially with own quality, with a slope greater than one, but then rises less steeply when ε exceeds 0.75. We may therefore compute the equilibrium premuneration value and dowry function as follows

$$\hat{U}(\varepsilon, r=2) = 0.5\varepsilon_2 + 0.5\phi(\varepsilon_1),$$

$$d(\varepsilon_1) = \frac{1}{4} - \frac{\varepsilon_1 + \sqrt{1-\varepsilon_1} - 1}{2}.$$

We see that the dowry always positive, but is non-monotone in quality. It is maximal for least and most attractive men, is minimal at $\varepsilon_1 = \frac{3}{4}$. Intuitively, men in the middle of moderate attractiveness experience an increase in the quality of their partner when the sex ratio increases, and thus have to be compensated less in the form of dowries, unlike men at the extreme ends of the attractiveness distribution.

To summarize, our model predicts a robust effect of changes in the effective sex ratio, r^*, upon the relative positions of the sexes. If r^* increases, the position of women worsens while that of men increases. The effects on dowries is more nuanced, since this depends upon how premuneration values change with r^*. If the main component of match value is the desire for marriage (i.e. the benefit an individual derives from being married as compared to being single), then premuneration values do not change. In consequence, the dowry must rise as r^* increases. However, more complex effects are possible when match value depends on multiple considerations, such as attractiveness and the desire for marriage.

5 Transitory shocks and non-steady state dynamics

We now consider a transitory shock, and show that dynamic adjustment is very different. In particular, the age gap will adjust away from τ^* for cohorts that are affected by the shock. The extent of adjustment is greater – and so the shock has less consequences for the affected cohorts – when age preferences are less important, as compared to quality considerations.

For simplicity, consider an infinite horizon economy with $g = 0$, so that there is no trend growth. Thus at every date, there are a unit measures of men and women. Let ε and η be distributed on $[0, 1]$ with cumulative distribution functions F and G respectively. Assume that preferences are additive and take the form:

$$Q(\varepsilon, \eta, \tau) = \varepsilon + \eta - h(\tau - \tau^*).$$

We assume that the h is non-negative and strictly convex, and equals 0 when $\tau - \tau^* = 0$. Let $c = h(-1)$.

Consider first the steady state allocation. Suppose that $F = G$ and $\bar{u} = \bar{v}$, so that the sexes are symmetric. Then it is easy to see that the marginal types are $\tilde{\varepsilon} = \tilde{\eta} = \bar{u}$, and equilibrium payoffs are $u^*(\varepsilon) = \max\{\varepsilon, \bar{u}\}$ and $v^*(\eta) = \max\{\eta, \bar{u}\}$.

To reduce notation, let us re-number the time indices of the men only, by adding τ^*, leaving the time indices of the women unaffected. Under this re-numbering, the ideal age gap is 0.

Consider now perturbing this economy so that at date 0, there is an unanticipated transitory shock so that the measure of women equals $r > 1$.[16] Now if $c = h(-1)$ is very large, the affected cohort of women will continue to have age gap τ^* only. The following equations describe the equilibrium conditions:

$$\tilde{\varepsilon}_o + \tilde{\eta}_0 = \bar{u} + \bar{v},$$

$$1 - F(\tilde{\varepsilon}_0) = r\left[1 - G(\tilde{\eta}_0)\right].$$

Let us assume that F and G are uniform on the unit interval, so that we can explicitly solve the above, yielding

$$\tilde{\eta}_0 = \frac{(r-1) + 2\bar{u}}{r+1} > \bar{u},$$

$$\tilde{\varepsilon}_0 = \frac{(r-1) + 2r\bar{u}}{r+1} < \bar{u}.$$

Thus the marginal type of woman increases in the affected cohort. Since cohort 1 is unaffected, stability requires that

$$\tilde{\eta}_0 + \tilde{\varepsilon}_1 - c \leq 2\bar{u} \iff \tilde{\eta}_0 + c \leq \bar{u} \iff c \geq \frac{(r-1)(1-\bar{u})}{r+1}. \tag{6}$$

The equilibrium payoff of a woman of type η in cohort t is given by

[16] For simplicity, we consider an increase in the number of women rather than a shock to cohort size – the effects are rather similar in both cases.

$$
\begin{aligned}
v_t(\eta) &= \bar{u} + \max\left\{\int_{\tilde{\eta}_t}^{\eta} Q_y(\varepsilon, y, \tau^*)dy, 0\right\} \\
&= \bar{u} + \max\{(\eta - \tilde{\eta}_t), 0\}.
\end{aligned}
$$

Since

$$
v_0(\eta) = \begin{cases} \bar{u} + (\eta - \tilde{\eta}_0) \text{ if } \eta \geq \tilde{\eta}_0 \\ \bar{u} \text{ if } \eta < \tilde{\eta}_0, \end{cases}
$$

we see that the distributional impact on women in the directly affected cohort is decreasing in $\tilde{\eta}_0$, which also equals the proportion of unmatched women in the cohort. The greater the proportion of unmatched women, the worse the position of the matched women. Conversely, since $u_0(\varepsilon) = \bar{u} + \max\{\varepsilon - \tilde{\varepsilon}_0, 0\}$, and since $\tilde{\varepsilon}_0$ and $\tilde{\eta}_0$ move in opposite directions, an increase in $\tilde{\eta}_0$ improves the position of men in cohort zero, who find themselves in demand due to the shock. These distributional effects apply more generally, when c is smaller, and when the shock changes age gaps and affects adjacent cohorts.

Now assume that c is smaller, so that the inequality 6 is violated, so that some adjustment in the age gap must take place. If c is just slightly lower than the above bound, then only cohort 1 will be affected. The system of equations is now given by

$$
\tilde{\varepsilon}_0 + \tilde{\eta}_0 = 2\bar{u},
$$

$$
\tilde{\varepsilon}_1 + \tilde{\eta}_1 = 2\bar{u},
$$

$$
[1 - F(\tilde{\varepsilon}_0)] + [1 - F(\tilde{\varepsilon}_1)] = r\,[1 - G(\tilde{\eta}_0)] + [1 - G(\tilde{\eta}_1)]\,.
$$

Solving this system, we find that

$$
\tilde{\eta}_0 = \frac{(r-1) + 4\bar{u} + 2c}{r+3} > \bar{u}.
$$

$$
\tilde{\varepsilon}_0 = \bar{u} - \tilde{\eta}_0 < \bar{u}.
$$

$\tilde{\eta}_1 = \tilde{\eta}_0 - c$ and $\tilde{\varepsilon}_1 = \tilde{\varepsilon}_0 + c$. When adjustment is spread out over two cohorts, the distributional effects in cohort 0 are smaller, i.e. the women are hurt less, and the men also benefit less, since the change in $\tilde{\eta}_0$ is smaller. For the adjustment to be restricted to two cohorts only, a necessary condition is that the marginal woman in cohort 1 (who is adversely affected), cannot do better by matching with a marginal man in the unaffected cohort, cohort

2:

$$\tilde{\varepsilon}_2 + \tilde{\eta}_1 - c \leq 2\bar{u} \Longleftrightarrow c \geq \frac{(r-1)(1-\bar{u})}{4+2r}.$$

For smaller values of c, the adjustment will be spread out over more cohorts. First, one can show that marrying couples will differ by index 0 or by index -1 – there cannot be, for example, a stable match where a woman of cohort zero is matched with a man of cohort 2. However, a woman of cohort 1 will be matched with such a man, so that age gap adjustment also takes place for women in cohorts that are not directly affected by the shock. This pattern of adjustment follows from the strict convexity of the cost function $h(.)$.

We now turn to solving for an equilibrium more generally. Let k denote the index of the largest affected cohort. The system of equations is as follows:

$$\tilde{\varepsilon}_t + \tilde{\eta}_t = 2\bar{u}, 0 \leq t \leq k.$$

$$\tilde{\varepsilon}_{t+1} + \tilde{\eta}_t - c = 2\bar{u}, 0 \leq t \leq k-1.$$

$$\sum_{t=0}^{k} [1 - F(\tilde{\varepsilon}_t)] = r\,[1 - G(\tilde{\eta}_0)] + \sum_{t=1}^{k} [1 - G(\tilde{\eta}_t)]$$

When both $F(.)$ and $G(.)$ are uniform on $[0,1]$, we can explicitly solve this system for the threshold values:

$$\tilde{\eta}_0 = \frac{(r-1) + 2(k+1)\bar{u} + k(k+1)c}{2k+1+r}.$$

$$\tilde{\varepsilon}_0 = \bar{u} - \tilde{\eta}_0.$$

$$\tilde{\eta}_t - \tilde{\eta}_{t+1} = c, 0 \leq t \leq k-1.$$

$$\tilde{\varepsilon}_{t+1} - \tilde{\varepsilon}_t = c, 0 \leq t \leq k-1.$$

$$\tilde{\eta}_k \leq \bar{u} + c.$$

The last inequality ensures that the marginal woman in cohort k, with quality greater than $\bar{u}$, cannot do better by matching with a man from cohort $k+1$. Since $\tilde{\eta}_k = \tilde{\eta}_0 - kc$, we may use this inequality to derive k : it equals the smallest non-negative integer greater than

$$\frac{(r-1)(1-\bar{u})}{c} - (1+r).$$

We see therefore that is decreasing in c. We summarize the effects of a transitory shock in the following proposition.

Proposition 4 *A transitory shock that increases the number of women in a single cohort will affect matching patterns in a finite number of cohorts,k, so that the age-gaps in the affected cohorts are τ^* or τ^*-1. If age considerations are less important so that c is smaller, then k is larger and the adjustment is spread over more cohorts, and women in cohort* 0 *(the directly affected cohort) are less adversely affected while men in cohort 0 benefit less. As* $c \to 0$, $k \to \infty$, *so that the adjustment is spread over very many cohorts, and has a very small effect on the directly affected cohort..*

6 Non transferable utility

Now let us consider the other polar extreme, of non-transferable utility. Since our model has two dimensions of preferences – quality and age – the non-transferable utility is hard to analyze with any degree of generality. If men and women have different ideal age gaps, then the vertical dimension, then the possibility of trading "quality" for the age gap dimension becomes complicated. To avoid this, we can consider two cases. First, when the quality dimension is absent, so that only the age gap matters for preferences. Second, when both women and men have the same ideal point, so that $\tau_B = \tau_G = \tau^*$.

Non transferable utility implies that the total payoff corresponding to a match, $Q(q(\varepsilon,\eta), S(\tau))$ can only be divided in a fixed way. Let $Q^m(q(\varepsilon,\eta), S(\tau))$ and $Q^f(q(\varepsilon,\eta), S(\tau))$ be functions such that

$$Q(q(\varepsilon,\eta), S(\tau)) = Q^m(q(\varepsilon,\eta), S(\tau)) + Q^m(q(\varepsilon,\eta), S(\tau)),$$

for every value of (ε,η,τ). Assume further that each sexes share in the share increases (weakly) as the total payoff increases.

Theorem 5 *Suppose that men and women have identical ideal age gaps, i.e. $\tau_B = \tau_G = \tau^*$. Then the unique steady stage age gap under stable matching is τ^*, and matching is assortative in the quality dimension, so that $\phi(\varepsilon)$ is increasing in η.*

Proof. Suppose that the age gap is τ^* and matching is assortative. If a man of type ε prefers a woman of type η and age gap τ', then $\eta > \phi(\varepsilon)$, and thus $\varepsilon < \phi^{-1}(\eta)$ and this women strictly prefers her current match to him. A similar argument applies to any woman. Now suppose that we have a steady state where $\tau \neq \tau^*$. Matching must be assortative – otherwise, if $\varepsilon > \varepsilon'$ and $\phi(\varepsilon) < \phi(\varepsilon')$, the pair $(\varepsilon, \phi(\varepsilon'))$ have a profitable deviation. Thus in every cohort, any man of type ε is matched to the same type of woman, $\phi(\varepsilon)$. Consider a pair of individuals of types $(\varepsilon, \phi(\varepsilon))$ and age gap τ^*. This pair can deviate from the proposed equilibrium matching, and increase both their payoffs. ■

Consider next the case where there is no vertical element of quality. Let $\bar{r}$ be given, and suppose that the equilibrium age gap is τ. Let r be the marriage market sex ratio corresponding to the pair $\bar{r}, \tau$; i.e. $r = (1+g)^{\tau}$. We claim that τ must equal the ideal point of the short side of the market. That is, if $r > 1$, then τ must equal τ_B, and if $r < 1, \tau = \tau_G$. To see this, let us suppose that $r > 1$. Then in every cohort, there are unmatched women. Thus a man can always propose to his ideal woman, i.e. one with an age gap of τ_B, and this proposal will be accepted since the woman is unmatched. This implies that any age gap τ different from τ_B cannot be stable. To see that τ_B is stable, note that no man can do better, and furthermore, any woman who is matched can also not do any better, since no age gap other than τ_B will be acceptable to a man. A similar argument establishes that the unique steady state stable match must equal τ_G when $r < 1$.

Consider finally the case where $r = 1$ so that the marriage market is balanced, e.g. because $\bar{r} = 1$ and $g = 0$ – this situation is a good approximation to marriage market conditions for much of human history. In this case, any age gap $\hat{\tau}$ that lies between τ_G and τ_B is an equilibrium steady state age gap. Consider a steady state where every man is matched to a woman with age gap $\hat{\tau}$. Since $r = 1$, every woman is also matched with age gap $\hat{\tau}$.Suppose that a man who proposes to woman he prefers – such a woman must have an age gap relative to him of τ' that is closer to τ_B than $\hat{\tau}$ is. However, since $\hat{\tau}$ lies between τ_B and τ_G, this implies that τ' is further away from this woman's ideal point τ_G than $\hat{\tau}$ is, and so this proposal is not acceptable to the woman.

Theorem 6 *There exists a stable steady state matching, for all parameter values. i) If $\bar{r}(1+g)^{\tau_G} > 1$ and $\bar{r}(1+g)^{\tau_B} > 1$, the unique equilibrium age gap equals τ_B. ii) If $\bar{r}(1+g)^{\tau_G} < 1$ and $\bar{r}(1+g)^{\tau_B} < 1$, the unique equilibrium age gap equals τ_G. iii) If neither the two above conditions hold, so that $\left(\bar{r}(1+g)^{\tau_G} - 1\right)\left(\bar{r}(1+g)^{\tau_B} - 1\right) \leq 0$, there exists a steady state where all women and all men are matched. In particular, if $\bar{r} = 1$ and $g = 0$, then every age gap $\hat{\tau}$ between τ_G and τ_B is a stable age gap.*

Proof. Parts (i) and (ii) have been proven in the text, so we turn to (iii). Let $\hat{\tau}$ be any value of τ that solves the equation $\bar{r}(1+g)^{\hat{\tau}} = 1$, so that if the age gap is $\hat{\tau}$, then the marriage market sex ratio r equals one. We show now that if $\hat{\tau}$ is an integer between τ_G and τ_B, then $\hat{\tau}$ will be a stable steady state age gap. Consider a steady state where every man is matched to a woman with age gap $\hat{\tau}$, and where every individual is matched – from the definition of $\hat{\tau}$ such a matching is measure preserving and thus feasible.Suppose that a man who proposes to woman he prefers – such a woman must have an age gap relative to him of τ' that is closer to τ_B than $\hat{\tau}$ is. However, since $\hat{\tau}$ lies between τ_B and τ_G, this implies that τ' is further away from this woman's ideal point τ_G than $\hat{\tau}$ is, and so this proposal is not acceptable to the woman. Suppose that no integer value of $\hat{\tau}$ that solves the equation $\bar{r}(1+g)^{\hat{\tau}} = 1$. Consider the expression

$$h(\lambda) \equiv \lambda\bar{r}(1+g)^{\tau_G} + (1-\lambda)\bar{r}(1+g)^{\tau_B} - 1.$$

Since h is continuous, $h(0) > 0$ and $h(1) < 0$, there exists a value $\hat{\lambda}$ such that $h(\hat{\lambda}) = 0$. Let every individual be matched, with a fraction $\hat{\lambda}$ in each cohort having an age gap τ_G with the remainder having an age gap τ_B. This matching is constructed to be measure preserving, and also, no individual is left unmatched. It is also stable, since no woman with matched age gap τ_G will accept a different match, and no man with age gap τ_B will also accept a different match. ■

This theorem illustrates why the age gap does not necessarily adjust endogenously in order to equilibrate demand and supply in the marriage market, even with non-transferable utility. Suppose that the initial situation is one where the sex ratio at birth is the normal one, i.e. $\bar{r} = 1$ and where $g = 0$. In this case, the equilibrium age gap will be some $\hat{\tau}$ between τ_B and τ_G. Now if g becomes positive, there will be an excess supply of girls, and so the equilibrium age gap will shift to τ_B. So the age gap increases, aggravating the marriage market imbalance if $\tau_B > \tau_G$, if men prefer a larger age gap than women. The age gap will decrease if $\tau_G > \tau_B$. Similarly, the effect of sex selection for males, that reduce r, is to make the age gap that preferred by women – this may reduce marriage market imbalance, or aggravate it.

To summarize, we have examined two leading models of the marriage market, transferable and non-transferable utility, and found that the age gap at marriage does not necessarily adjust in order to reduce the systematic marriage market imbalance arising from the marriage squeeze or biases in the sex ratio at birth. The underlying reasons for the age gap in these models are individual preferences – this seems a plausible foundation given the overwhelming evidence from over 90 countries, that the age gap is always positive. An alternative explanation for the age gap is provided by Bergstrom and Bagnoli (1990), who set out a signalling model. Men are privately informed about their ability, and the more able ones delay marriage, in order to secure a better quality match. While a complete discussion of their model is beyond the scope of this paper, we have analyzed the implications of changing marriage market sex ratios upon the age gap in their model, and also find that gap does not necessarily adjust in order to reduce marriage market imbalance.

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