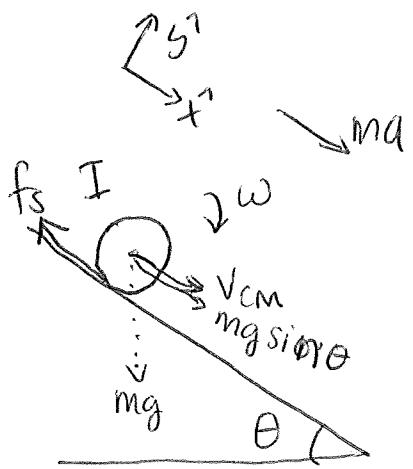


Wentworth Institute of Technology
 Engineering Physics I - Problem Set 10 SOLUTIONS

1)



f_s : static friction, keeps object in pure roll

$$\Sigma F_x = ma = mg \sin \theta - f_s$$

$$\vec{r}_{cm} = \vec{r} \times \vec{f}_s = R f_s = I \alpha$$

$$\alpha = a/R \text{ pure roll so } f_s = \frac{I \alpha}{R} = \frac{I a}{R^2}$$

$$ma = mgs \sin \theta - \frac{I a}{R^2}$$

$$ma + \frac{I a}{R^2} = mgs \sin \theta \rightarrow \boxed{a = \frac{mgs \sin \theta}{\left(m + \frac{I}{R^2}\right)}}$$

b) thin walled hollow cylinder $I = MR^2$

$$\rightarrow a = \frac{mgs \sin \theta}{m + \frac{MR^2}{R^2}} = \frac{mgs \sin \theta}{2m} \rightarrow \boxed{a = \frac{1}{2} g \sin \theta}$$

c) solid cylinder $I = \frac{1}{2}MR^2$

$$a = \frac{mgs \sin \theta}{m + \frac{\frac{1}{2}MR^2}{R^2}} = \frac{mgs \sin \theta}{\frac{3}{2}m} \rightarrow \boxed{a = \frac{2}{3} g \sin \theta}$$

$$d) \text{ a thin walled hollow cyl. } = \frac{1}{2} g \sin \theta \quad I_{\text{thincyl.}} = MR^2$$

$$\text{A solid cylinder } = \frac{2}{3} g \sin \theta \quad I_{\text{solid cyl.}} = \frac{1}{2} MR^2$$

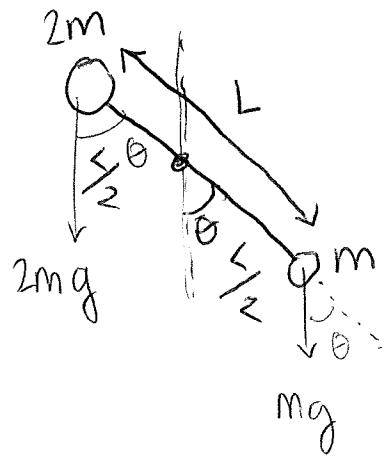
The ^{net} acceleration for the two objects is not the same because the static friction force depends on I .

$(f_s = \frac{I\alpha}{R^2})$ This tells us that the force required to accelerate the ^{rotational}/_{angular} motion (α) is larger for larger I .

(This is similar to saying that the force required to accelerate an object in ^{translational} linear motion depends on its mass, $F=ma$) For the rolling cylinders, the acceleration also does not depend on the objects' masses but does depend on the distribution of mass.

$$2) \text{ a)} I = \sum m_i r_i^2$$

$$I = 2m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \boxed{\frac{3}{4}mL^2 = I}$$



$$\vec{r} = \vec{r}_x \hat{i}$$

$$= +\frac{L}{2}(2mg)\sin\theta + -\frac{L}{2}(mg)\sin 180^\circ$$

$$= +Lmg\sin\theta + -\frac{L}{2}mg\sin\theta$$

$$T = +\frac{L}{2}mg\sin\theta \quad \text{out of page } \hat{i}$$

$$T = I\alpha = \frac{L}{2}mg\sin\theta \overset{\hat{i}}{=} \frac{3}{4}mL^2\alpha$$

$$\boxed{\frac{2}{3}\frac{g\sin\theta}{L} = \alpha} \quad \text{out of page } \hat{i}$$

$$3) T = 25 \text{ days} = \frac{2\pi r}{v} \rightarrow v = \frac{2\pi r}{T} = wr \rightarrow \omega = \frac{2\pi}{T}$$

$$\text{conservation of } \vec{L}: I_1\omega_1 = I_2\omega_2$$

$$\frac{2}{5}M_1R_1^2\omega_1 = \frac{2}{8}M_2R_2^2\omega_2 \quad M_1=M_2$$

$$(750,000 \text{ km})^2 \frac{2\pi}{25 \text{ days}} = (10^4 \text{ km})^2 \frac{2\pi}{T_2}$$

$$3) \frac{(750000)^2}{25 \text{ days}} = \frac{(10^4)^2}{T_2} \quad T_2 \text{ in days}$$

$$T_2 = \left(10^8 / (750000)^2\right) 25 \text{ days} = .00444 \text{ days}$$

$\times \frac{(24 \times 60) \text{ min}}{1 \text{ day}}$

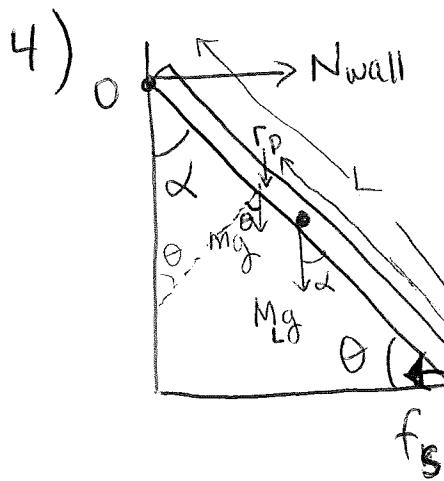
$$\boxed{T_2 = 6.39 \text{ minutes}}$$

b) $KE_{\text{rot}} = \frac{1}{2} I w^2$

$$\omega_1 = \frac{2\pi}{25 \text{ days}} \quad \omega_2 = \frac{2\pi}{6.39 \text{ min}} = \frac{2\pi}{.00444 \text{ days}}$$

$$\omega_1 \ll \omega_2 \quad \text{so} \quad \frac{1}{2} I \omega_1^2 \ll \frac{1}{2} I \omega_2^2$$

so $\boxed{KE_{\text{rot}} \text{ increases}}$



how far can a person of mass m climb? (without ladder moving)

Ladder mass M_L

There must be a normal force acting on the ladder from both the wall and the ground. We can see that there must

be a normal force from the wall because otherwise $T_{\text{point P at ground}} \neq 0$

$$(T_p = 0 = +\frac{L}{2} M_L g \underbrace{\sin \alpha}_{\cos \theta} - L N_{\text{wall}} \sin \theta \quad \text{without person on it})$$

let position of person be r_p along ladder measured from ground

$$T_p = \frac{L}{2} M_L g \underbrace{\sin \alpha}_{\cos \theta} + r_p m g \underbrace{\sin \alpha}_{\cos \theta} - L N_{\text{wall}} \sin \theta = 0$$

$$0 = T_{\text{o(at wall)}} = -(L - r_p) m g \underbrace{\sin(90 + \theta)}_{180 - \alpha} - \frac{L}{2} M_L g \underbrace{\sin \alpha}_{\cos \theta} + L f_s \sin \theta + L N_g \underbrace{\sin \alpha}_{\cos \theta}$$

$$T_{\text{cm}} = -\frac{L}{2} N_{\text{wall}} \sin \theta + \left(r_p - \frac{L}{2}\right) m g \sin \alpha - \frac{L}{2} f_s \sin \theta + \frac{L}{2} N_g \underbrace{\sin \alpha}_{\cos \theta} = 0$$

$$T_p = \frac{L}{2} M_L g \cos \theta + r_p mg \cos \theta - L N_{\text{wall}} \sin \theta$$

$$\sum F_y: mg + M_L g = N_g \quad \sum F_x: f_s = N_{\text{wall}}$$

$$r_p mg \cos \theta = L N_{\text{wall}} \sin \theta - \frac{L}{2} M_L g \cos \theta$$

$$r_p = \frac{L f_s \sin \theta - \frac{L}{2} M_L g \cos \theta}{mg \cos \theta}$$

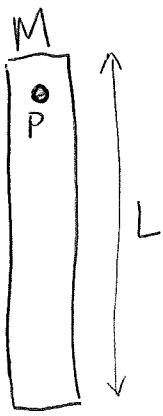
Sanity check: $r_p \downarrow$ as θ ($\cos \theta$) \downarrow

since $f_s = \mu_s N = \mu_s (m + M_L) g$ we can rewrite r_p :

$$r_p = \frac{L M_L (\mu_s \sin \theta - \frac{1}{2} \cos \theta) + L \mu_s m \sin \theta}{m \cos \theta}$$

$$\text{if } M_L = 0 \quad r_p = L \mu_s \tan \theta$$

5)

 $\begin{matrix} \text{y} \\ \text{x} \end{matrix}$ m, v_0 

completely inelastic - two bodies move with common velocity after collision

a) conservation of \vec{L} : (no external torques)

$$\begin{aligned}\vec{L}_{p_1} &= \vec{r} \times \vec{p}_1 \quad \text{right before collision} \\ &= +L M V_0 \hat{z}\end{aligned}$$

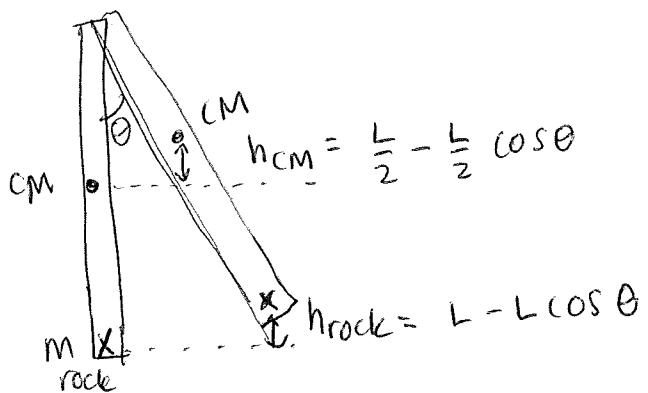
$$\vec{L}_2 = I_2 \omega_2 = L M V_0$$

$$= \left(\frac{1}{3} M L^2 + m L^2 \right) \omega_2 = L M V_0$$

$$\omega_2 = \frac{L M V_0}{L^2 \left(\frac{1}{3} M + m \right)} = \boxed{\frac{M V_0}{\frac{1}{3} M + m}} = \omega \quad (\hat{z})$$

\vec{L} is conserved at point P immediately before + after the collision because there are no net torques - Mg goes through CM and is \parallel to \vec{r} from P

b)



$$\frac{1}{2} I \omega^2 = m_{\text{rock}} g h_{\text{rock}} + M_{\text{CM}} g h_{\text{CM}}$$

$$\frac{1}{2} (L^2) \left(\frac{1}{3} M + m \right) \omega^2 = (L - L \cos \theta) g m_r$$

$$+ \frac{1}{2} (L - L \cos \theta) g M_{\text{CM}}$$

5b)

$$\frac{1}{2} L^2 \left(\frac{1}{3} M + m \right) \omega^2 = L(1 - \cos \theta) \left(g m_{\text{rock}} + \frac{1}{2} g M \right)$$

$$\frac{1}{2} L^2 \left(\frac{1}{3} M + m \right) \frac{m^2 v_0^2}{L^2 \left(\frac{1}{3} M + m \right)^2} = L(1 - \cos \theta) g \left(m + \frac{1}{2} M \right)$$

$$\frac{1}{2} \frac{\left(\frac{1}{3} M + m \right) m^2 v_0^2}{\left(\frac{1}{3} M + m \right)^2} \frac{1}{L g \left(m + \frac{1}{2} M \right)} = 1 - \cos \theta$$

$$\cos \theta = 1 - \frac{m^2 v_0^2}{2 \left(\frac{1}{3} M + m \right) L g \left(m + \frac{1}{2} M \right)}$$

$$\boxed{\theta = \cos^{-1} \left[1 - \frac{m^2 v_0^2}{2 \left(\frac{1}{3} M + m \right) L g \left(m + \frac{1}{2} M \right)} \right]}$$