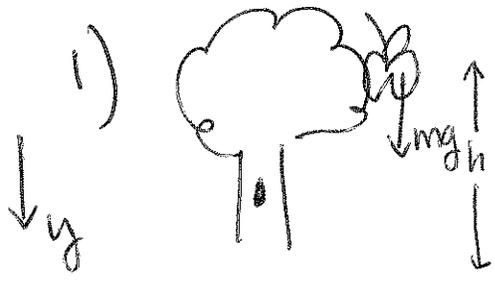


Wentworth Institute of Technology
 Engineering Physics I
 Problem set 6 - SOLUTIONS

6



$$W = \int \vec{F} \cdot d\vec{s} = \vec{F} \cdot \vec{s}$$

$$\vec{F} = mg \hat{y}$$

$$\vec{s} = h \hat{y}$$

$$\boxed{W = mgh}$$

b) work-energy theorem $W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, $v_f =$ final ~~velocity~~ speed
 $v_i = 0$ so $W = \frac{1}{2}mv_f^2 = mgh$

$$\boxed{v_f = \sqrt{2gh}}$$

c) t_{hit} : $y(t) = h = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$

$$h = \frac{1}{2}gt^2 \rightarrow \cancel{t = \sqrt{2gh}} \quad t = \sqrt{\frac{2h}{g}}$$

$$v(t) = v_0 + a_y t$$

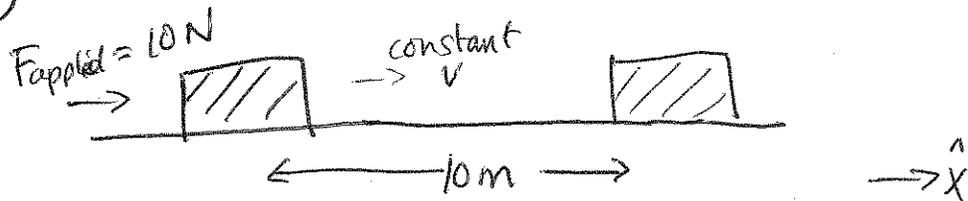
$$v(t = \sqrt{\frac{2h}{g}}) = v_f = v_0 + gt$$

$$v_f = g \sqrt{\frac{2h}{g}} = \boxed{\sqrt{2gh} = v_f} \quad \checkmark \quad \text{Same as part b!}$$

d) final speed does not depend on mass

A dropped watermelon would have the same final speed.
 Gravitational force would be same on the watermelon (mgh)

2) $m = 5 \text{ kg}$ $\mu_k = 0.2$ $g = 10 \text{ m/s}^2$



a) work I do:

$$W = \int \vec{F} \cdot d\vec{s} = \vec{F} \cdot \vec{s}$$

$$\vec{F} = 10 \text{ N } \hat{x}$$

$$\vec{s} = 10 \text{ m } \hat{x}$$

$$\vec{F} \cdot \vec{s} = 10 \text{ N} \cdot 10 \text{ m} = \boxed{100 \text{ N} \cdot \text{m} = W_{\text{applied force}}}$$

b) $F_{\text{friction}} = \mu_k m g = 0.2 (5 \text{ kg}) (10 \text{ m/s}^2) = 10 \text{ N } \hat{x}$

($F_{\text{friction}} = F_{\text{applied}}$ because const. velocity)

$$W = \vec{F} \cdot \vec{s} = -10 \text{ N } \hat{x} \cdot 10 \text{ m } \hat{x} = \boxed{-100 \text{ N} \cdot \text{m} = W_{\text{friction}}}$$

c) net work done on the box = $W_{\text{friction}} + W_{\text{applied force}} = 0 \text{ (N} \cdot \text{m)}$

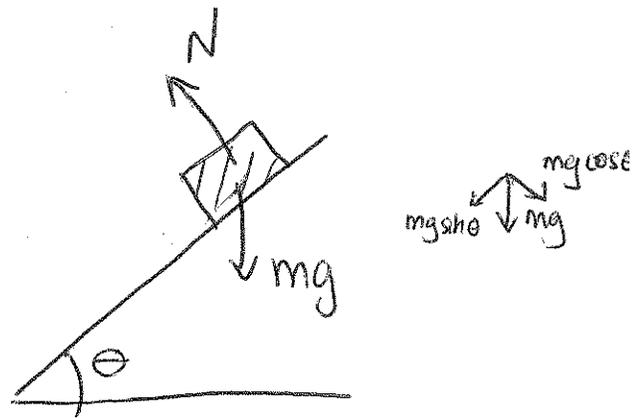
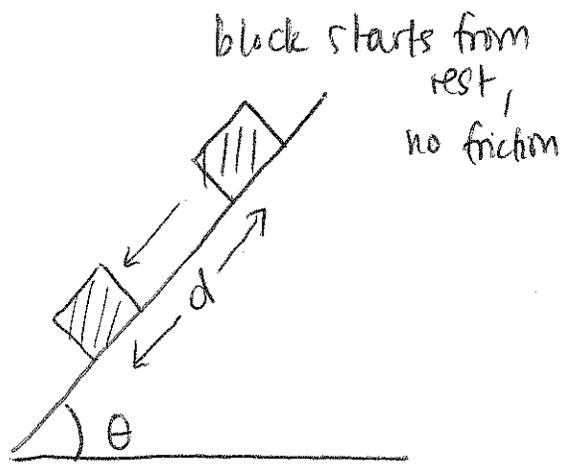
- or - net work = $\vec{F}_{\text{net}} \cdot \vec{s}$

but $\vec{F}_{\text{net}} = 0$ because velocity is constant

(we can ignore the gravitational force +

Normal force - they do no work b/c they are \perp to motion)

3)



- a) My intuition says that the final speed should not depend on mass but it may depend on θ .
 If there is a θ dependence I would expect final speed to increase w/ larger θ

- b) What is v_{final} ?

The only two forces on the block are mg and N
 N does no work b/c it is \perp to \vec{s}

$$\text{work done by gravitational force} = \vec{F} \cdot \vec{s}$$

$$\vec{F} = mg \sin \theta \hat{x} + mg \cos \theta \hat{y}$$

$$\vec{s} = d \hat{x}$$

$$\vec{F} \cdot \vec{s} = (mg \sin \theta \hat{x} + mg \cos \theta \hat{y}) \cdot (d \hat{x} + 0 \hat{y})$$

$$W = (d)(mg \sin \theta) + 0$$

$$W = \Delta KE = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$\frac{1}{2} m v_f^2 = d m g \sin \theta \rightarrow v_f^2 = \frac{2 d g \sin \theta}{\cdot}$$

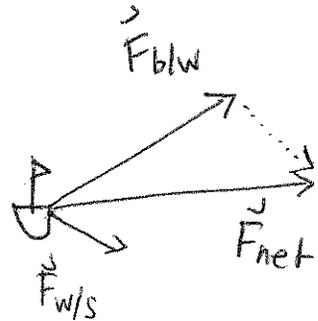
$$v_f = \sqrt{2 d g \sin \theta}$$

3c) My intuition was correct regarding mass.

I find that v_f is related to θ and does $\uparrow w / \uparrow \theta \checkmark$

$$4) \quad \vec{F}_{\text{boat/water}} = 6 \text{ N } \hat{x} + 4 \text{ N } \hat{y}$$

$$\vec{F}_{\text{water/shore}} = 2 \text{ N } \hat{x} - 1 \text{ N } \hat{y}$$



$$a) \quad \text{net force} = \vec{F}_{b/w} + \vec{F}_{w/s} = \vec{F}_{\text{boat/shore}}$$

$$\vec{F}_{\text{boat/shore}} = \vec{F}_{b/w} + \vec{F}_{w/s}$$

$$= (6+2) \text{ N } \hat{x} + (4-1) \text{ N } \hat{y}$$

$$= \boxed{8 \text{ N } \hat{x} + 3 \text{ N } \hat{y} = \vec{F}_{\text{net}}}$$

b) Boat travels along \vec{F}_{net} (by Newton's 2nd law)

$$\vec{s} = 8 \hat{x} + 3 \hat{y} \text{ (m)} \quad , \quad |\vec{s}| = \sqrt{64+9} = \sqrt{73}$$

if the boat travels a total of 100 m then the ^{vector} distance traveled

$$\text{is given by } \vec{d} = \frac{8 \cdot 100}{\sqrt{73}} \hat{x} + \frac{3 \cdot 100}{\sqrt{73}} \hat{y} \text{ meters}$$

$$\begin{aligned} \text{work done by wind} &= \vec{F}_{b/w} \cdot \vec{d} = (6 \hat{x} + 4 \hat{y}) \cdot \left(\frac{800}{\sqrt{73}} \hat{x} + \frac{300}{\sqrt{73}} \hat{y} \right) \\ &= \frac{4800}{\sqrt{73}} + \frac{1200}{\sqrt{73}} = \frac{6000}{\sqrt{73}} = \boxed{702 \text{ J}} \end{aligned}$$

$$\begin{aligned}
 \text{c) } W_{\text{in boat by river}} &= \vec{F}_{r/s} \cdot \vec{d} = (2\hat{x} - 1\hat{y}) \cdot \left(\frac{800}{\sqrt{73}}\hat{x} + \frac{300}{\sqrt{73}}\hat{y} \right) \\
 &= \frac{1600}{\sqrt{73}} - \frac{300}{\sqrt{73}} = \frac{1300}{\sqrt{73}} = \boxed{152 \text{ J}}
 \end{aligned}$$

N
N
N
N

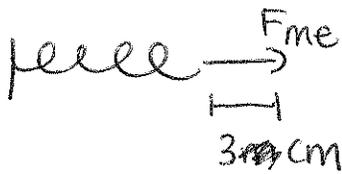
N·m
N·m

$$\begin{aligned}
 \text{d) total work} &= W_{\text{wind}} + W_{\text{river}} = 702 \text{ J} + 152 \text{ J} \\
 &= \boxed{854 \text{ J}}
 \end{aligned}$$

$$\begin{aligned}
 \text{- or - } W_{\text{total}} &= \vec{F}_{\text{net}} \cdot \vec{d} \\
 &= (8\hat{x} + 3\hat{y}) \cdot \left(\frac{800}{\sqrt{73}}\hat{x} + \frac{300}{\sqrt{73}}\hat{y} \right) \\
 &= \frac{6400}{\sqrt{73}} + \frac{900}{\sqrt{73}} = \frac{7300}{\sqrt{73}} = \boxed{854 \text{ J}}
 \end{aligned}$$

N·m

5) 12 J of work = 1 do to stretch a spring
3 cm



$$a) \quad F = kx \quad W = \int \vec{F} \cdot d\vec{x} = \frac{1}{2} k \Delta x^2 = \frac{k(3\text{cm})^2}{2} = 12 \text{ J}$$

$$= \int kx dx =$$

$$12 \text{ J} = \frac{k(0.03 \text{ m})^2}{2}$$

$$k = \frac{2(12 \text{ J})}{(0.03)^2 \text{ m}^2} = 2.67 \times 10^4 \frac{\text{N}}{\text{m}}$$

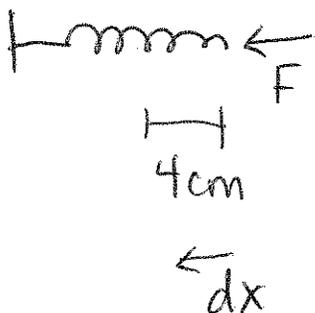
$$b) \quad F = kx$$

$$= (2.67 \times 10^4 \text{ N}) \cdot (0.03 \text{ m})$$

$$= \boxed{800 \text{ N} = F} \quad \text{force I applied}$$

$$\boxed{k = 2.67 \times 10^4 \text{ N/m}}$$

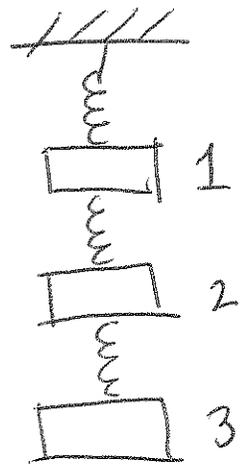
$$c) \quad W = \int \vec{F} \cdot d\vec{x} = \int +kx dx = \frac{1}{2} k \Delta x^2 = \frac{1}{2} (2.67 \times 10^4 \frac{\text{N}}{\text{m}}) (0.04 \text{ m})^2$$



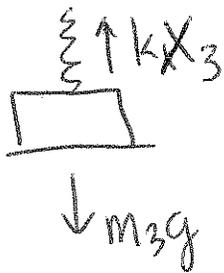
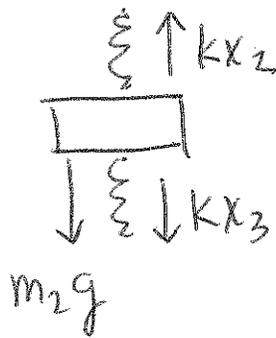
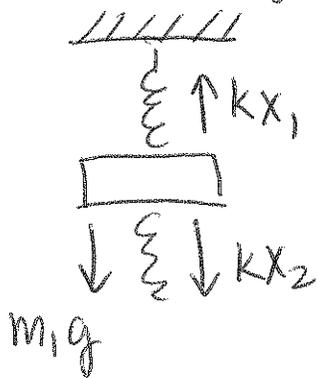
$$\boxed{W = 21.3 \text{ N} \cdot \text{m or J}}$$

work needed to compress
Spring 4 cm

- 6) 3 masses, all mass $m = 10 \text{ kg}$
 $k = 8 \text{ kN/m}$ all 3 springs
 original length all springs = 12 cm



a) Free body diagrams:



b) in equilibrium $\Sigma F = 0$

$$\Sigma F_{3y}: kx_3 = m_3g$$

$$\Sigma F_{2y}: kx_2 = kx_3 + m_2g = (m_3 + m_2)g$$

$$\Sigma F_{1y}: kx_1 = m_1g + kx_2 = (m_1 + m_2 + m_3)g$$

$$(b) \quad kx_3 = m_3 g, \quad m_3 = m_2 = m_1 = 10 \text{ kg}, \quad k = 8000 \text{ N/m}$$

$$x_3 = \frac{(10 \text{ kg})(9.81 \text{ m/s}^2)}{8000 \text{ N/m}} = 0.012 \text{ m}$$

$$kx_2 = (m_2 + m_3)g$$

$$x_2 = \frac{(20 \text{ kg})(9.81 \text{ m/s}^2)}{8000 \text{ N/m}} = 0.024 \text{ m}$$

$$kx_1 = (m_1 + m_2 + m_3)g$$

$$x_1 = \frac{30 \text{ kg} (9.81 \text{ m/s}^2)}{8000 \text{ N/m}} = 0.036 \text{ m}$$

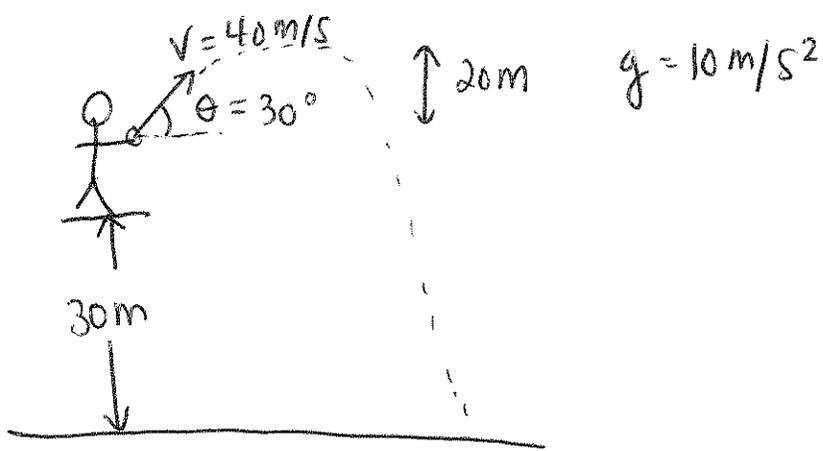
$$\text{Spring 3 is } 0.12 \text{ m} + 0.012 \text{ m} = 0.132 \text{ m} \quad (13.2 \text{ cm})$$

$$2 \text{ is } 0.12 \text{ m} + 0.024 \text{ m} = 0.144 \text{ m} \quad (14.4 \text{ cm})$$

$$1 \text{ is } 0.12 \text{ m} + 0.036 \text{ m} = 0.156 \text{ m} \quad (15.6 \text{ cm})$$

Spring 1 extends the most because it supports the most weight

Q7)



$$\text{max height} = 20 \text{ m} + 30 \text{ m} = 50 \text{ m}$$

$$(20 \text{ m} = h = \frac{v_0^2 \sin^2 \alpha}{2g} \quad \text{max height eq. from kinematics})$$

a) Using kinematics:

- ball's speed at max height is v_{ox}
 b/c there is no v_y at max. height and
 $a_x = 0$ so $v_x = v_{ox}$ at all times

$$V = v_{ox} = v_0 \cos 30^\circ = 40 \text{ m/s} \cos 30^\circ = \boxed{34.6 \frac{\text{m}}{\text{s}}}$$

- ball's speed when it hits the ground

first find t_{hit} : $y(t_{\text{hit}}) = 0 = 30 \text{ m} + 40 \frac{\text{m}}{\text{s}} \sin 30^\circ t - \frac{1}{2} g t_{\text{hit}}^2$

$(y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2)$

Quadratic equation for t $\rightarrow t_{\text{hit}} = \frac{-40 \text{ m/s} \sin 30^\circ \pm \sqrt{(40 \text{ m/s} \sin 30^\circ)^2 - 4(30 \text{ m})(\frac{1}{2} g)}}{-2(\frac{1}{2} g)}$

$$7b) t_{hit} = \frac{-40 \sin 30^\circ \text{ m/s} \pm \sqrt{40^2 \sin^2 30^\circ \text{ m/s}^2 - 4(30 \text{ m})(-\frac{1}{2}(10 \text{ m/s}^2))}}{2(-\frac{1}{2} 10 \text{ m/s}^2)}$$

$$t_{hit} = 5.16 \text{ s}$$

$$V_y(t_{hit}) = V_{0y} - g t_{hit} = 40 \sin 30^\circ \text{ m/s} - 10 \frac{\text{m}}{\text{s}^2} (5.16 \text{ s})$$

$$V_y = -31.62$$

$$V_x = V_0 \cos \theta = 34.6 \text{ m/s}$$

Speed when hits ground

$$|V| = \sqrt{\frac{(31.62)^2}{\text{m/s}} + (34.6 \text{ m/s})^2} = \underline{46.87 \text{ m/s} = V_{hit}}$$

c) ball's speed at max ht. using cons. of energy:

$$\frac{1}{2} m v_i^2 + m g y_i = \frac{1}{2} m v_h^2 + m g y_h$$

$$\frac{(40 \text{ m/s})^2}{2} + (10 \text{ m/s}^2)(30 \text{ m}) - (10 \text{ m/s}^2)(50 \text{ m}) = \frac{1}{2} v_h^2$$

$$\frac{1}{2} v_h^2 = 600 \frac{\text{m}^2}{\text{s}^2} \rightarrow v_h^2 = 1200 \text{ m}^2/\text{s}^2$$

$$v_h = \sqrt{1200 \text{ m}^2/\text{s}^2}$$

same as
part b ✓

$$\boxed{v_h = 34.64 \text{ m/s}}$$

7c) ball's speed when it hits the ground

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f^0$$

$$\frac{1}{2}(40\text{ m/s})^2 + 10\text{ m/s}^2(30\text{ m}) = \frac{1}{2}v_f^2$$

$$1100\text{ m}^2/\text{s}^2 = \frac{1}{2}v_f^2$$

$$v_f = \sqrt{2200\frac{\text{m}^2}{\text{s}^2}} = \boxed{46.9\text{ m/s} = v_f}$$

✓ same as part b