

Wentworth Institute of Technology

Engineering Phys. I

Problem Set 3 - REVISED SOLUTIONS

1) Planetary orbits

$$V = \frac{2\pi R}{T}, \quad a = \frac{V^2}{R}$$

Assume a
due to
Gravity

$$a = \frac{C}{R^2}$$

C unknown
constant

$$V_{\text{Earth}} = \frac{2\pi (1 \text{ AU})}{1 \text{ year}} = 2\pi \text{ AU/year}$$

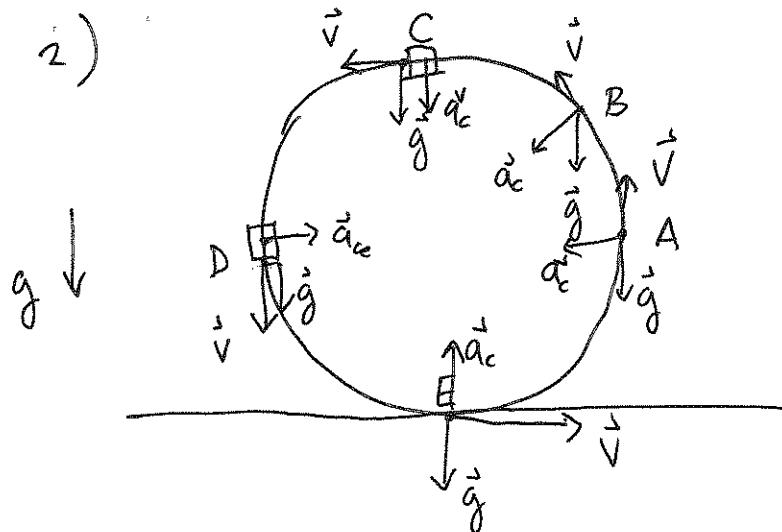
$$\frac{V_E^2}{R_E} = \frac{C}{R_E^2} \rightarrow C = V_E^2 R_E = 4\pi^2 \text{ AU}^3/\text{year}^2$$

For planets generally, $V_{\text{planet}}^2 = \frac{C}{R_{\text{planet}}}$

$$T_{\text{planet}} = \frac{2\pi R_{\text{planet}}}{V_{\text{planet}}}$$

	$R (\text{AU})$	$V (\text{AU/year})$	$T (\text{years})$
Mercury	.39	10.06	0.24
Venus	.72	7.4	0.61
Earth	1	6.28	1
Mars	1.52	5.09	1.87
Jupiter	5.2	2.75	11.85
Saturn	9.58	2.03	29.65
Uranus	19.23	1.43	84.3
Neptune	30.1	1.14	165.1

2)



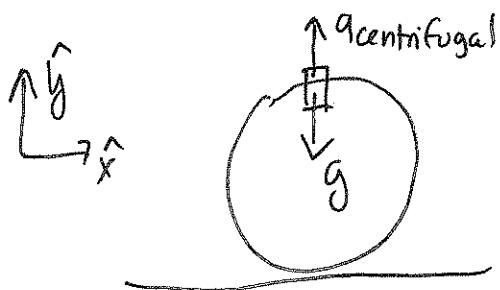
a) Non uniform circular motion because

$$a_{\parallel} \neq 0$$

$|\vec{v}|$ is not constant,

$$V_C < V_B < (V_D = V_A) < V_E$$

c) Minimum velocity needed:



let $a_{\text{centrifugal}}$ just balance gravity ($-\hat{y}$)

$$|a_{\text{centrifugal}}| = |a_{\text{centripetal}}| = \frac{v^2}{R}$$

$$g = \frac{v^2}{R} \rightarrow v^2 = gR$$

$$\boxed{v = \sqrt{gR}}$$

$$3) \frac{V^2}{R} = a = 12.5g \quad g = 9.81 \text{ m/s}^2 \quad R = 8.84 \text{ m}$$

a) want $f = \frac{1}{T}$ revolutions $\frac{\text{time}}{\text{frequency}}$

$$T = \frac{2\pi R}{V} \rightarrow V = \frac{2\pi R}{T}$$

$$a = \frac{V^2}{R} = \frac{4\pi^2 R^2}{T^2 R} = \frac{4\pi^2 R}{T^2} \rightarrow T^2 = \frac{4\pi^2 R}{a}$$

$$T^2 = \frac{4\pi^2 (8.84 \text{ m})}{(12.5) 9.81 \text{ m/s}^2}, \quad T^2 = 2.84 \text{ s}^2$$

$$T = 1.68 \text{ s} \times \frac{1 \text{ min}}{60 \text{ s}} = 0.028 \text{ min}$$

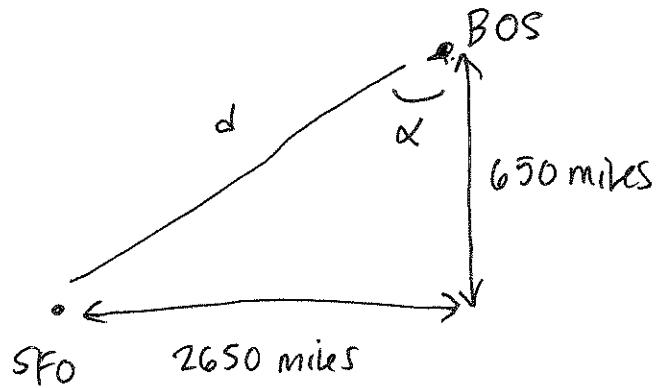
(*)

$$f = \frac{1 \text{ rev}}{0.028 \text{ min}} = \boxed{\underbrace{35.5 \frac{\text{rev}}{\text{min}}}_{} = f}$$

$$b) T^2 = \frac{4\pi^2 R}{20g} \Rightarrow T = \sqrt{\frac{4\pi^2 8.84 \text{ m}}{(20) 9.81 \text{ m/s}^2}} \Rightarrow T = 1.3 \text{ s} \\ = 0.22 \text{ min}$$

$$f = \frac{1}{0.22 \text{ min}} = \boxed{\underbrace{45 \frac{\text{rev}}{\text{min}}}_{} = f}$$

4)



engine thrust:

$$V_{\text{plane/air}} = 500 \text{ miles/hour}$$

a) $d^2 = 2650^2 + 650^2$ all units miles

$$\boxed{d = 2728 \text{ miles}}$$

b) $t = \frac{d}{v} = \frac{2728 \text{ miles}}{500 \text{ miles/hour}} = \boxed{5.45 \text{ hours}}$

$$V_{\text{south}} = V \cos \alpha$$

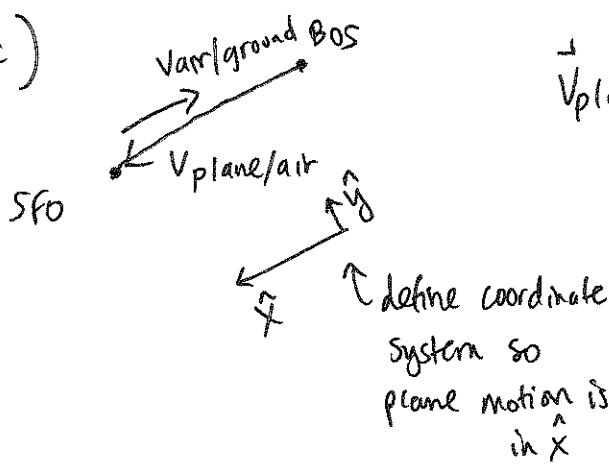
$$\tan \alpha = \frac{2650 \text{ mi}}{650 \text{ mi}} \rightarrow \alpha = 76.21^\circ$$

$$V_{\text{west}} = V \sin \alpha$$

$$V_{\text{south}} = \left(500 \frac{\text{miles}}{\text{hour}}\right) \cos 76^\circ = 119.1 \text{ miles/hour}$$

$$V_{\text{west}} = \left(500 \frac{\text{miles}}{\text{hour}}\right) \sin 76^\circ = 485.6 \text{ miles/hour}$$

c)



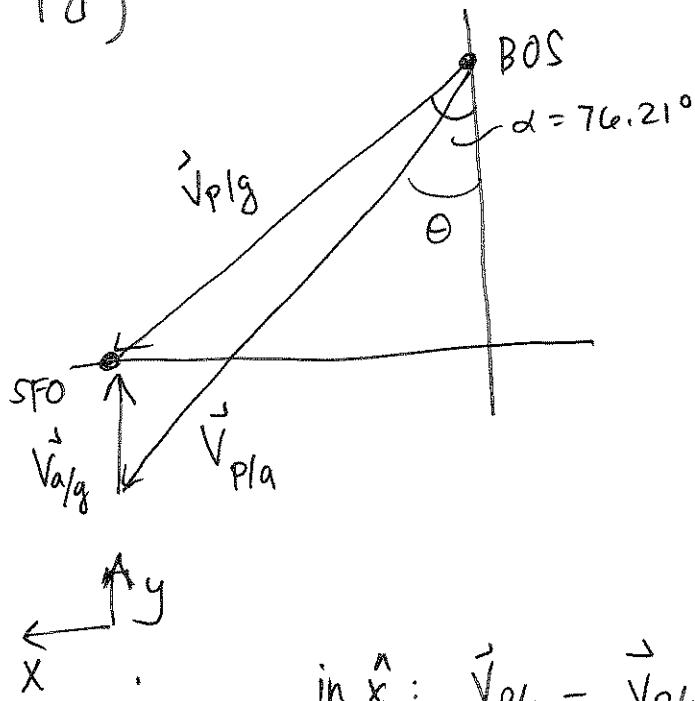
$$\vec{V}_{\text{plane/ground}} = \vec{V}_{\text{plane/air}} + \vec{V}_{\text{air/ground}}$$

$$= \underline{500 \frac{\text{miles}}{\text{hour}}} \hat{x} + \underline{(-30)} \frac{\text{miles}}{\text{hour}} \hat{x}$$

$$\vec{V}_{\text{plane/g}} = 470 \frac{\text{miles}}{\text{hour}} \hat{x}$$

$$t = \frac{d}{v} = \frac{2728 \text{ miles}}{470 \frac{\text{miles}}{\text{hour}}} = \boxed{5.8 \text{ hours}}$$

4d)



unknown V , magnitude of plane wrt g

pilot sets $V_{p/a}$ @ unknown angle θ , magnitude 500 miles/hour

$$\vec{V}_{p/g} = \vec{V}_{p/a} + \vec{V}_{a/g}$$

in \hat{x} : $\vec{V}_{p/g} = \vec{V}_{p/a}$

$$\boxed{V \sin 76.21^\circ = 500 \text{ miles/hour} \sin \theta}$$

in \hat{y} : $\boxed{V \sin 76.21^\circ = 500 \text{ miles/hour} \cos \theta - 25 \text{ miles/hour}}$

$$\begin{aligned} |V_{p/g}| &= V^2 = 500^2 \sin^2 \theta + (500 \cos \theta - 25)^2 \\ &= 500^2 \sin^2 \theta + 500^2 \cos^2 \theta - 2(25)500 \cos \theta + 25^2 \end{aligned}$$

$$V^2 = 500^2 + 25^2 - 50(500 \cos \theta)$$

$$500 \cos \theta = V \sin 76.21^\circ + 25$$

$$V^2 = 500^2 + 25^2 - 50(V \sin 76.21^\circ + 25)$$

$$= 500^2 + 25^2 - 50(25) - 50V \sin 76.21^\circ$$

$$V^2 + (50 \sin 76.21^\circ) V - 249375 = 0$$

$$\rightarrow V = 475.68 \text{ miles/hour}$$

solve for V
using quadratic formula

V_x of plane wrt air :

$$V \sin 76.21^\circ = 500 \frac{\text{miles}}{\text{hour}} \sin \theta$$

$$\theta = 67.5^\circ$$

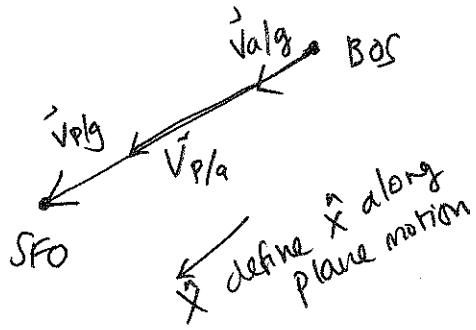
$$V_x = 500 \sin \theta = \boxed{398.9 \text{ miles/hour west}}$$

V_y of plane wrt air :

$$\Phi 500 \cos \theta = \boxed{301.3 \frac{\text{miles}}{\text{hour}} \text{ South}}$$

$$\boxed{\vec{V}_{P/a} = 398.9 \hat{x} + 301.3 \hat{y}} \quad (\hat{y} \text{ defined north})$$

4) e



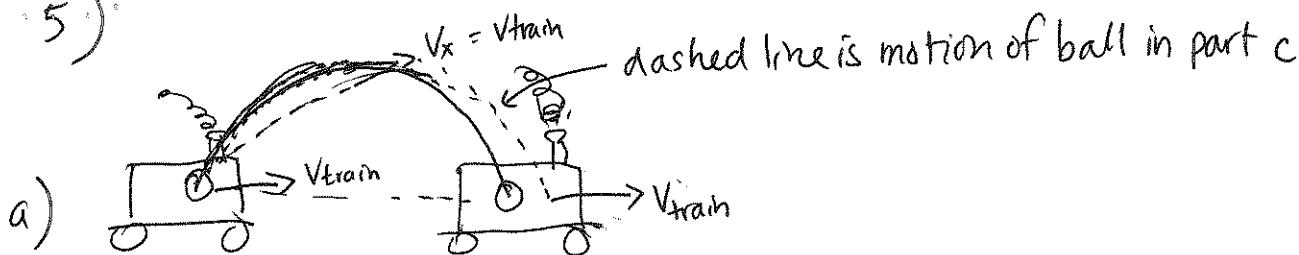
$$V_{min} = \frac{d}{P/g} = \frac{2728 \text{ miles}}{7 \text{ hours}} = 389.7 \frac{\text{miles}}{\text{hour}}$$

$$V_{pl/g} = V_{pl/a} + V_{a/g}$$

$$389.7 \frac{\text{miles}}{\text{hour}} = V_{pl/a} + 40 \frac{\text{miles}}{\text{hour}}$$

$$\boxed{V_{pla} = 349.7 \frac{\text{miles}}{\text{hour}}}$$

5)

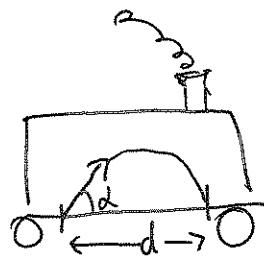


a)

An observer on the train will see the ball move up + down vertically (no horizontal motion)

An observer on the platform will see parabolic motion because of the ball (+ train's) initial x velocity

b)



Inside train, ball thrown at V_{ball} , d train distance to fire hd is d

$$t_d = \frac{2 V_0 \sin \alpha}{g}$$

V_0 is V_{ball}

(see lecture notes or past homework for derivation)

distance traveled ^{as seen by} observer = $d +$ distance train travels in t_d

$$d_{obs.} = d + (V_{train})(t_d)$$

Observer is given by ~~total obs. = total obs.~~

$$(V_{train} + V_{ball} \cos \alpha_{train}) t_d$$

$$V_{total\ obs.} \cos \alpha_{obs.} \cdot t_d = d_{obs.}$$

$$V_{total\ obs.}^2 = (V_{train} + V \cos \alpha_{train})^2 + (V_{ball} \sin \alpha_{train})^2$$

$$5b). V_{\text{total obs}} = V_{\text{train}} + V_{\text{ball}}^2 + 2V_{\text{train}}V_{\text{ball}} \cos \alpha_{\text{train}}$$

5b)
c)

revised

$$\text{for } V_{\text{train}} = 100 \text{ m/s}$$

$$V_{\text{ball}} = 10 \text{ m/s}$$

$$\alpha_{\text{train}} = 45^\circ$$

d cannot be 2m as specified because

$$d = \frac{V_0^2 \sin 2\alpha}{g} = \frac{(10 \frac{\text{m}}{\text{s}})^2 (\sin 90^\circ)}{9.81 \text{ m/s}^2} \approx 10 \text{ m}$$

so using eq. Specified w/ correct d:

$$t_d = \frac{2V_{\text{ball}} \sin \alpha_{\text{train}}}{g} = 1.445$$

$$d_{\text{obs}} = d + V_{\text{train}} t_d = 10 \text{ m} + 100 \text{ m/s} (1.445) = 154 \text{ m}$$

$$V_{\text{total obs}} = 107.3 \text{ m/s}, \cos \alpha_{\text{obs}} = \frac{d_{\text{obs}}}{t_d \cdot V_{\text{total obs}}} = \frac{154 \text{ m}}{(1.445 \cdot 107.3 \text{ m/s})}$$

$$\alpha_{\text{obs}} = \cos^{-1} \frac{154 \text{ m}}{154.3} \rightarrow \alpha_{\text{obs}} = 3^\circ$$