Engineering Physics I Fall 2015 – Makeup Midterm 1

Name: SOLUTIONS

You have 1 hour and 15 minutes to complete this exam.
You may not use your notes, textbook, or calculators.

Read all questions carefully and show your work as much as possible for partial credit. Circle your final answers. Be careful with units!

Remember these problem solving tips:

- *Include units in your answer and check that your units make sense*
- *Compare your answer to your intuition*
- *Draw a picture of the scenario described in the problem*

There are 4 problems. Good luck!

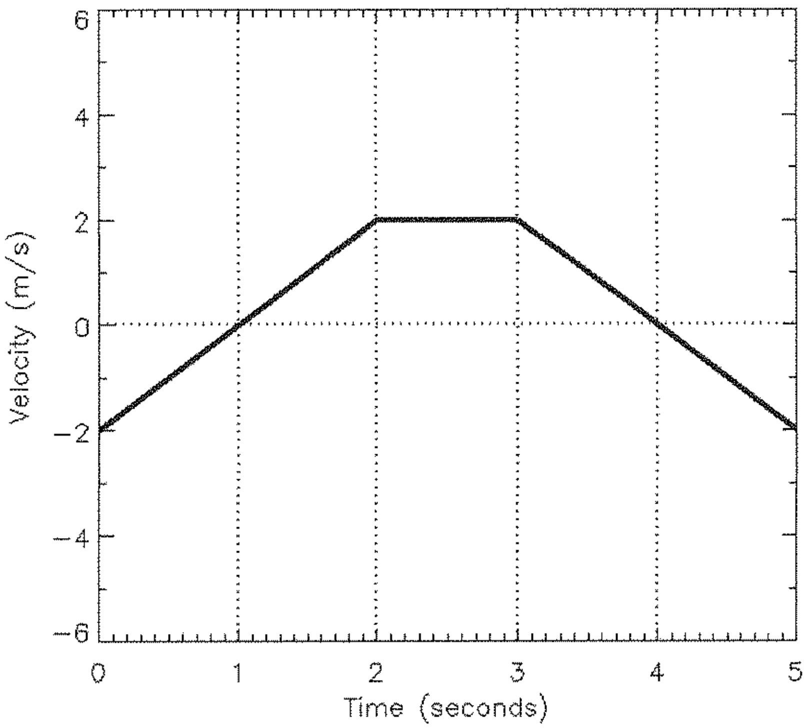
Where appropriate, you may express your answers in terms of square roots or trig functions or inverse trig functions (sin, cos, tan).

For some problems you may wish to use the following equations:

$$\vec{V}_{a/b} = \vec{V}_{a/c} + \vec{V}_{c/b}$$

$$a_c = \frac{v^2}{R}$$

Problem 1 (30 points)



A particle moves in the x direction with no velocity or acceleration in the y direction. At time t = 0, x = 0. The velocity *v* of the particle changes with time as shown in the figure.

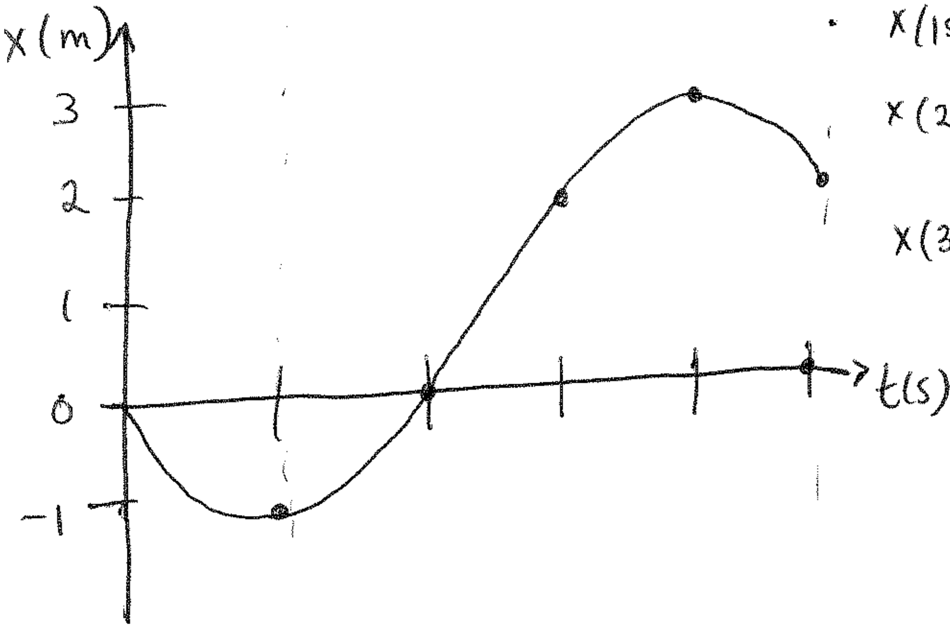
a) Draw x(t) over the same time range. Label your axes!
b) What is the acceleration at t=1s and t=4s?
c) What is the average speed and average velocity between t=0 and t=5 seconds?

@ $t=1s$ $a = \frac{\Delta v}{\Delta t} = \frac{2 - -2 \, m/s}{2s}$

$a = 2 m/s^2$

@ $t=4s$, $a = -2 m/s^2$

Can apply $x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to each interval w/ const. accel. $\begin{pmatrix} 0\to 2s \\ 2\to 3s \\ 3\to 4s \end{pmatrix}$



$x(1s) = 0 - 2\frac{m}{s}(1s) + \frac{1}{2}(2 m/s^2)(1s) = -1m$

$x(2s) = 0 - 2\frac{m}{s}(2s) + \frac{1}{2}(2\frac{m}{s^2})(2s)^2 = -4m + 4m$
$= 0m$

$x(3s) = 0m + 2\frac{m}{s}(1s) + 0 = 2m$

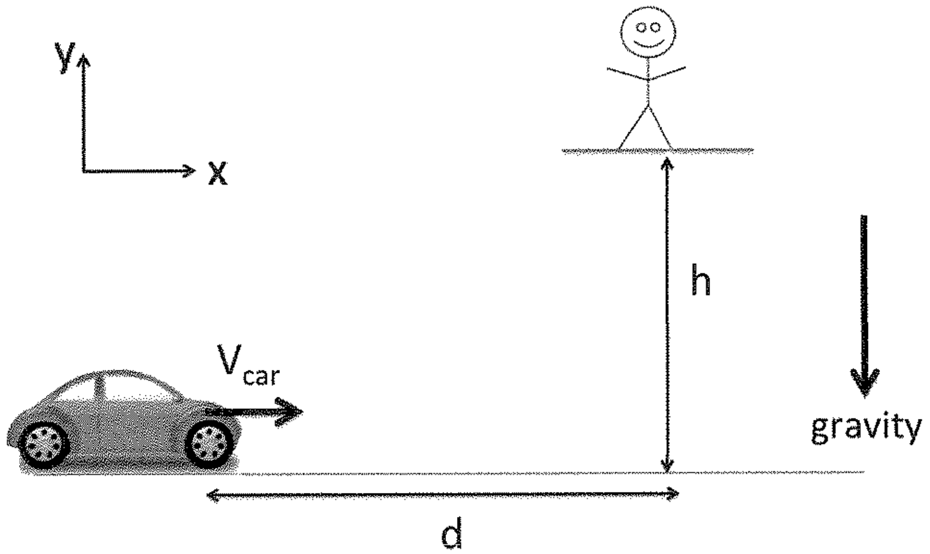
$x(4s) = 2m + 2\frac{m}{s}(1s) + \frac{1}{2}(-2\frac{m}{s^2})(1s)^2$
$= 4m - 1m = 3m$

$x(5s) = 2m + 2\frac{m}{s}(2s) + \frac{1}{2}(-2\frac{m}{s^2})(2s)^2$
$= 2m + 4m - 4m = 2m$

c) Average velocity $= \frac{\Delta x}{\Delta t} = \frac{2m}{5s} = \boxed{0.4 m/s \, \hat{x}}$

avg. speed $= \frac{1}{5s}(1+1+2+1+1 \, m) = \boxed{\frac{6}{5} m/s}$

Problem 2 (20 points)



At time t=0s a person is at rest, located a height h above the ground and a distance d from a car that is moving toward the person as shown with a velocity v_{car}. At time t=0s, the person throws a ball up in the air with an initial velocity of $\mathbf{v_{ball}} = v_{ball}\,\mathbf{y}$. The ball's velocity is just right so that the ball will hit the car.

a) When will the ball hit the car?

b) Derive an expression for v_{ball} in terms of h, d, v_{car}, and g. What does it mean if $\frac{gd^2}{2v_{car}^2} - h < 0$ (or, to put it another way, $h > \frac{gd^2}{2v_{car}^2}$) ?

a) ball will hit car when both have same x position

$x_{ball}(t) = d$ at all times $\quad x_{car}(t) = 0 + V_{car}t$

$$d = V_{car}t \rightarrow \boxed{t = \frac{d}{V_{car}}}$$

b) $y_{ball}(t) = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$, at $t = d/V_{car}$ $y(t) = 0$

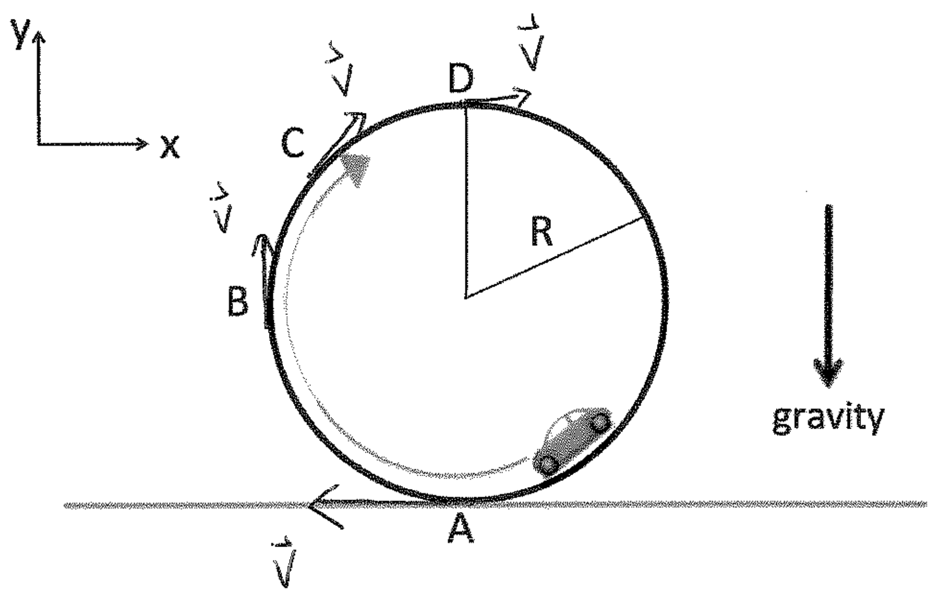
$$0 = h + V_{ball}t - \frac{1}{2}gt^2$$

$$0 = h + V_{ball}\frac{d}{V_{car}} - \frac{1}{2}g\frac{d^2}{V_{car}^2} \rightarrow \frac{gd^2}{2V_{car}^2} - h = V_{ball}\frac{d}{V_{car}}$$

$$\boxed{V_{ball} = \frac{d}{V_{car}}\left(\frac{gd^2}{2V_{car}^2} - h\right)}$$

if $\frac{gd^2}{2V_{car}^2} - h < 0$ then V_{ball} is negative - person must begin by throwing ball down $(-\hat{y})$ otherwise the ball will miss the car

Problem 3 (20 points)



A toy car is driving counter clockwise on a circular track of radius $R = 1$ m as shown. Let $g = 10\ m/s^2$.

a) Draw the car's velocity vectors (taking care with the magnitude of each vector) at points A, B, C, and D. Is this uniform or non-uniform circular motion?

b) At point D the car experiences a centripetal acceleration of 100 cm/min². Convert this acceleration into m/s² then calculate the car's velocity (in m/s) at point D.

c) If the track vanishes when the car is at point D, how far does the car travel in the x direction before it hits the ground?

a) non-uniform circular motion - gravity has a component // to motion so $|\vec{v}|$ is not constant

b) $\frac{100\,\cancel{cm}}{\cancel{min^2}} \times \frac{1\,m}{100\,\cancel{cm}} \times \frac{1\,\cancel{min}}{60\,s} \times \frac{1\,\cancel{min}}{60\,s} = \boxed{\frac{1}{3600}\frac{m}{s^2} = a}$, $a = \frac{v^2}{R}$ $\quad v = \sqrt{aR}$

$$v = \sqrt{\frac{1}{3600}\frac{m}{s^2}\cdot(1\,m)} = \boxed{\frac{1}{60}\frac{m}{s} = v}$$

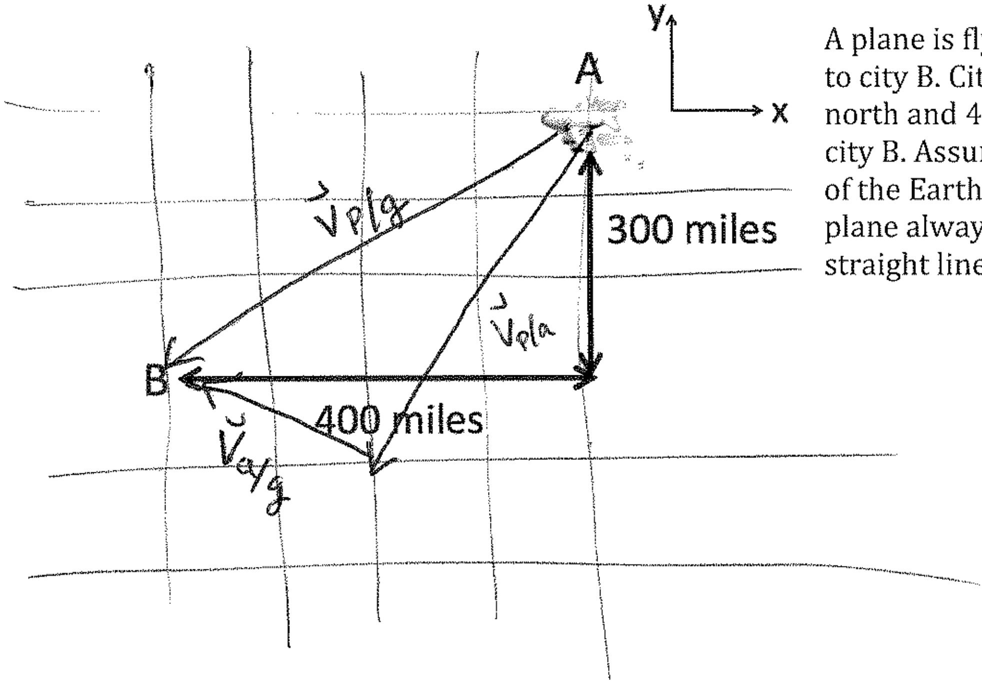
c) $v_x = \frac{1}{60}\frac{m}{s}$, $y(t_{ground}) = 0 = 2R + \cancel{v_{0y}t} - \frac{1}{2}g t_{ground}^2$ $\quad R = 1m$

$2R = \frac{1}{2} g t_{gr.}^2$ $\quad \frac{4m}{g} = t^2$, $t = \frac{2}{\sqrt{10}}s$

$x(t_{gr}) = \cancel{x_0} + v_{0x}t + \cancel{\frac{1}{2}a_x t^2}$

$= \frac{1}{60}\frac{m}{s}\cdot\frac{2}{\sqrt{10}}s = \boxed{\frac{1}{30\sqrt{10}}\ m = x}$

Problem 4 (30 points)



A plane is flying from city A to city B. City A is 300 miles north and 400 miles east of city B. Assume the surface of the Earth is flat and the plane always travels in a straight line.

a) If there is no wind (velocity of air with respect to the ground), what is the plane's velocity (express as a vector) with respect to the ground?

b) Because of winds, the plane takes off and travels with a velocity (with respect to the air) of roughly 100 miles per hour west and 200 miles per hour south. To an observer on the ground, the plane flies from city A to city B in a straight line and the journey takes 2 hours. What is the magnitude and direction of the wind velocity? For the magnitude, it is okay if you do not fully complete the calculation – just show me that you have set it up correctly. (Hint: What is the velocity of the plane with respect to the ground?)

c) Draw and label the relevant vectors from part b on the figure above. each grid marker is 50 mph

a) $\vec{V}_{p/g} = -400\text{ mph}\,\hat{x} - 300\text{ mph}\,\hat{y}$ no wind

b) $\vec{V}_{p/a} = -100\text{ mph}\,\hat{x} - 200\text{ mph}\,\hat{y}$

$\vec{V}_{p/g} = \vec{V}_{p/a} + \vec{V}_{a/g}$

$\vec{V}_{p/g} = -\frac{400\text{ miles}}{2\text{ hours}}\hat{x} - \frac{300\text{ miles}}{2\text{ hours}}\hat{y}$

$\vec{V}_{a/g}$ (wind) $= \vec{V}_{p/g} - \vec{V}_{p/a}$

$= -200\text{ mph}\,\hat{x} - 150\text{ mph}\,\hat{y}$

wind velocity $\vec{V}_{a/g} = (-200 - {}^{-}100)\text{mph}\,\hat{x} + (-150 - {}^{-}200)\text{mph}\,\hat{y} = -100\text{ mph}\,\hat{x} + 50\text{ mph}\,\hat{y}$

$\boxed{\vec{V}_{a/g} = -100\text{ mph}\,\hat{x} + 50\text{ mph}\,\hat{y}}$, $|\vec{V}_{a/g}| = \sqrt{100^2 + 50^2}$ mph