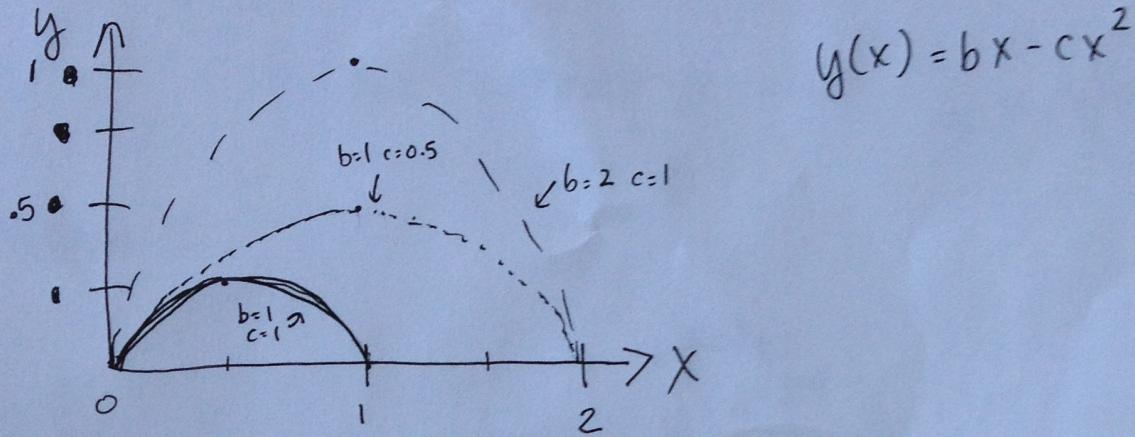


Wentworth Institute of Technology
Engineering Physics I
Problem set 2 solutions



Parabolic motion

~~Reads more related to basic physics~~

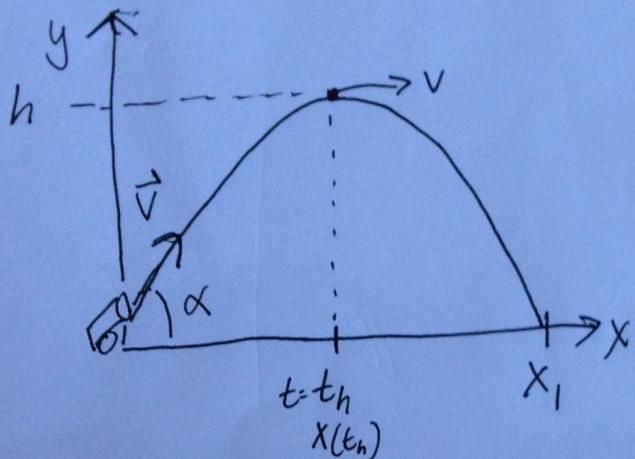
From graphs above, b seems to be related to the angle α at which an object is projected (w.r.t. horizontal) and c seems to be related to how far the object travels (so its initial velocity) in x

From class/your textbook we know

that parabolic motion actually follows

$$y(x) = (\tan \alpha) x - \left(\frac{g}{2V_0^2 \cos^2 \alpha} \right) x^2$$

Problem 2



initially:

$$v_y = v_0 \sin \alpha$$

$$v_x = v_0 \cos \alpha$$

$$v_{y,0} = v_0 \sin \alpha$$

$$v_{x,0} = v_0 \cos \alpha$$

My intuition says:

- if $v \uparrow$ by factor of 2 : h will increase (by $\sqrt{2}$?)
 x_i will increase (by a factor of 2?)
- if $v \downarrow$ by factor of 2 : h will decrease
 x_i will decrease
- v stays the same but $\alpha \uparrow$: h will increase
 x_i will decrease

What is h ?

We know that at $y = h$, $v_y = 0$ and let time at highest point be called t_h

So using $v_y(t) = v_{y,0} + a_y t$

$$v_y(t_h) = v_0 \sin \alpha - g t_h = 0 \quad , \quad t_h = \frac{v_0 \sin \alpha}{g}$$

→ solve for t_h

and plug into eq. for $y(t)$

$$y(t) = y_0 + v_{0,y} t + \frac{1}{2} a_y t^2 \rightarrow \text{generic}$$

$$y(t_h) = y_0 + v_{0,y} t_h + \frac{1}{2} a_y t_h^2 \rightarrow \text{specific to } t_h$$

$$h = 0 + v_0 \sin \alpha t_h - \frac{1}{2} g t_h^2$$

$$h = v_0^2 \frac{\sin^2 \alpha}{g} - \frac{1}{2} \left(v_0^2 \frac{\sin^2 \alpha}{g^2} \right) = \boxed{\frac{v_0^2 \sin^2 \alpha}{2g} = h}$$

so if $v \uparrow$ by factor of 2,

$h \uparrow$ by factor of 4

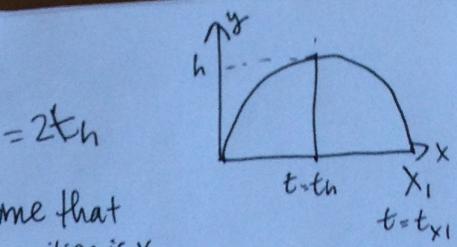
$v \downarrow$ by factor of 2, $h \downarrow$ by ~~4~~ ^{factor of} 4

$\alpha \uparrow$ then $h \uparrow$

What is x_1 ?

Since motion is parabolic, we know $t_{x_1} = 2t_h$

We can plug t_{x_1} into $x(t)$:



at time that
x position is x_1

$$x(t) = x_0 + v_{0,x} t + \frac{1}{2} a_x t^2 \rightarrow \text{generic}$$

$$x(t_{x_1}) = x_0 + v_{0,x} t_{x_1} + \frac{1}{2} a_x t_{x_1}^2$$

$$x_1 = 0 + v_0 \cos \alpha t_{x_1} + 0 \quad \text{no accel. in x-direction}$$

$$x_1 = \frac{2 v_0^2 \cos \alpha \sin \alpha}{g}$$

$$t_{x_1} = 2t_h = \frac{2 v_0 \sin \alpha}{g}$$

use trig. identity

$$2 \cos x \sin x = \sin 2x$$

$$\boxed{x_1 = \frac{v_0^2 \sin 2\alpha}{g}}$$

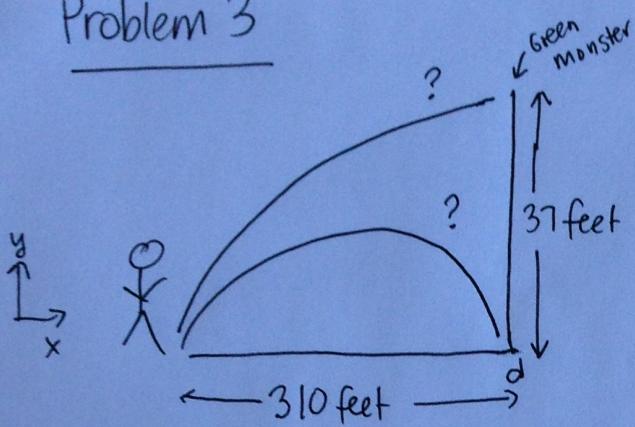
so if $v \uparrow$ by a factor of 2, $x_1 \uparrow$ by factor of 4

$$v \downarrow \quad " \quad , x \downarrow \quad "$$

if α increases (and is less than 45°) it depends!

$$\begin{aligned} \alpha = 20^\circ &\rightarrow 2\alpha = 40^\circ &] \text{doubling } \alpha \text{ increases } x_1 \\ \sin 2\alpha = \sin 40^\circ &< \sin 2(2\alpha) = \sin 80^\circ & \text{from } 20^\circ \\ x_1 \text{ for: } &x_1 \text{ if } \alpha \text{ doubles:} \\ \alpha = 40^\circ &\rightarrow 2\alpha = 80^\circ &] \text{doubling } \alpha \text{ decreases } x_1 \\ \sin 2\alpha = \sin 80^\circ &> \sin 2(2\alpha) = \sin 160^\circ = \sin 20^\circ & \end{aligned}$$

Problem 3



$$V_y = V_0 \sin \alpha$$

$$V_x = V_0 \cos \alpha$$

$$\alpha = 30^\circ$$

$$V_0 = 85 \text{ miles/hour}$$

$$37 \text{ feet} = 11.2 \text{ m}$$

$$310 \text{ feet} = 94.4 \text{ m}$$

$$85 \frac{\text{miles}}{\text{hour}} = 37.99 \text{ m/s}$$

a) Will the ball reach the Green Monster?

$$\underset{\text{Max x distance}}{d} = \frac{V_0^2 \sin 2\alpha}{g} = \frac{(38 \frac{\text{m}}{\text{s}})^2 \sin 60^\circ}{9.81 \frac{\text{m}}{\text{s}^2}}$$

$$d = 127.4 \text{ m}$$

$d > 94.4 \text{ m}$ (distance to Green Monster)

So yes, it will reach it
(hit it or go over)

b) Will the ball make it over the wall for a home run?

let t at which $x = 310$ feet be t_{wall}

we know: $x(t_{\text{wall}}) = 310 \text{ feet} = x_0 + v_{0,x} t_{\text{wall}} + \frac{1}{2} a_x t_{\text{wall}}^2$

if we solve for t_{wall} in terms of things we know, we can plug

that into $y(t)$ to see if $y(t_{\text{wall}}) > 37 \text{ feet } (11.2 \text{ m})$

$$x(t_{\text{wall}}) = 310 \text{ feet} = 94.4 \text{ m} = x_0 + v_0 \cos \alpha t_{\text{wall}} + \frac{1}{2} a_x t_{\text{wall}}^2$$

$$94.4 \text{ m} = 0 + v_0 \cos \alpha t_{\text{wall}} + 0$$

$$\frac{94.4 \text{ m}}{(38 \frac{\text{m}}{\text{s}}) \cos 30^\circ} = \frac{94.4 \text{ m}}{v_0 \cos 30^\circ} = t_{\text{wall}}$$

$$2.86 \text{ s} = t_{\text{wall}}$$

$$\begin{aligned}y(t_{\text{wall}}) &= y_0 + v_{0,y} t + \frac{1}{2} a_y t^2 \\&= 0 + v_0 \sin \alpha t_{\text{wall}} - \frac{1}{2} g t_{\text{wall}}^2 \\&= (38 \frac{\text{m}}{\text{s}}) \left(\frac{1}{2}\right) (2.86 \text{ s}) - \frac{1}{2} (9.81 \frac{\text{m}}{\text{s}^2}) (2.86 \text{ s})^2\end{aligned}$$

$$y(t_{\text{wall}}) = 14.21 \text{ m}, \text{ wall height is } 11.2 \text{ m}$$

so yes the ball will make it over the wall

c) What is the minimum velocity ($\alpha = 30^\circ$ still) for a home run?

from part b we derived $t_{\text{wall}} = \frac{x(t_{\text{wall}})}{v \cos \alpha}$ (generic)

$$t_{\text{wall}} = \frac{94.4 \text{ m}}{v \cos 30^\circ} \quad (\text{specific to problem, } v \text{ unknown})$$

and: $y(t_{\text{wall}}) = v \sin \alpha t_{\text{wall}} - \frac{1}{2} g t_{\text{wall}}^2$ v still unknown

Set $y(t_{\text{wall}}) = 11.2 \text{ m}$ and solve for v :

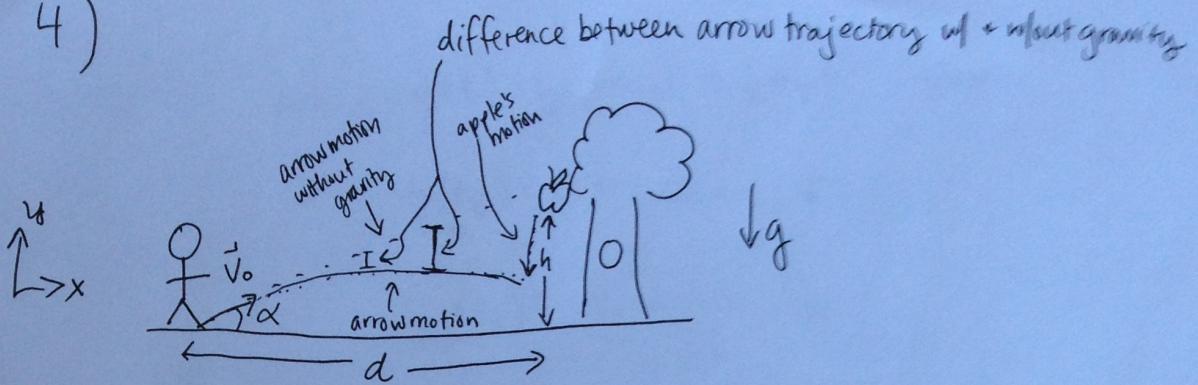
$$11.2 \text{ m} = v \sin 30^\circ t_{\text{wall}} - \frac{1}{2} g t_{\text{wall}}^2$$

$$11.2 \text{ m} = \underbrace{\cancel{v \sin 30^\circ} \frac{94.4 \text{ m}}{\cancel{v \cos 30^\circ}}}_{54.5 \text{ m}} - \frac{1}{2} g \left(\frac{94.4 \text{ m}}{v^2 \cos^2 30^\circ} \right)$$

$$-43.3 \text{ m} \doteq -\frac{1}{2} (9.81 \frac{\text{m}}{\text{s}^2}) \frac{94.4 \text{ m}^2}{v^2 (0.75)} \rightarrow v^2 = \frac{(9.81 \frac{\text{m}}{\text{s}^2})(94.4)^2 \text{ m}^2}{2 (0.75) (-43.3 \text{ m})}$$

$$\boxed{V = 36.6 \text{ m/s}}$$

4)



Apple falls at same time arrow is released.

Will the arrow hit the apple?

We assume the arrow reaches $x=d$ before the apple hits the ground

At any time t the arrow and the apple have fallen the same distance relative to where they would be without gravity.

(Without gravity the apple would be in the tree + the arrow would aim right for it.)

Eq. of motion for the apple: $x(t)=d$

$$y(t) = y_0 + v_{0,y}t + \frac{1}{2}a_y t^2 \quad (\text{generic})$$

$$y(t) = \bullet h + 0 - \frac{1}{2}gt^2 = h - \frac{1}{2}gt^2$$

Eq. of motion for the arrow: $x(t) = x_0 + v_{0,x}t + \frac{1}{2}a_x t^2$ generic
(no x accel.)

let t_d be the time the arrow gets to $x=d$

$$x(t_d) = v_0 \cos \alpha t_d = d \rightarrow t_d = \frac{d}{v_0 \cos \alpha}$$

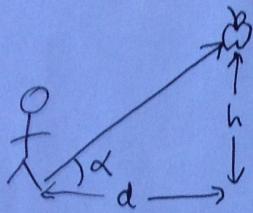
We can plug t_d into $y(t)$ to see where in y the arrow is at time t_d when $x=d$

$$4 \text{ contd}) \quad \text{arrow} \quad y(t) = y_0 + v_0 t + \frac{1}{2} a y t^2 \quad \text{generic}$$

$$t_d = \frac{d}{v_0 \cos \alpha}$$

$$\text{arrow} \quad y(t_d) = 0 + v_0 \sin \alpha t_d - \frac{1}{2} g t_d^2$$

$$\text{arrow} \quad y_{\text{arrow}}(t_d) = \frac{v_0 \sin \alpha d}{v_0 \cos \alpha} - \frac{g d^2}{2 v_0^2 \cos^2 \alpha}$$



$$\boxed{y_{\text{arrow}}(t_d) = d(\tan \alpha - \frac{g d}{2 v_0^2 \cos^2 \alpha})}$$

We have a hit if:

$$y_{\text{apple}}(t_d) = y_{\text{arrow}}(t_d), \quad \text{and we can relate } h+d \text{ by } \tan \alpha = \frac{h}{d} \text{ (see figure)}$$

$$y_{\text{apple}}(t_d) = h - \frac{1}{2} g t_d^2 = d \tan \alpha - \frac{g}{2} t_d^2$$

$$= d \tan \alpha - \frac{g}{2} \frac{d^2}{v_0^2 \cos^2 \alpha}$$

$$\boxed{y_{\text{apple}}(t_d) = d \left(\tan \alpha - \frac{g d}{2 v_0^2 \cos^2 \alpha} \right)}$$

$$y_{\text{apple}}(t_d) = y_{\text{arrow}}(t_d) \quad \checkmark$$