The Battle of the Sexes, Dynamic Version

- **Vanesa moves first**: she buys a ticket *either* for the football match *or* for the opera.

- She shows Miguel her ticket, so he knows what she has done.

- **Then Miguel moves**: he buys his ticket *either* for the football match *or* for the opera.

- The game in normal form:
A Different Game Model

- Yes, \( \langle R, \text{Always } R \rangle \) is a Nash equilibrium, but it will not occur if both players are rational.

- This is because Miguel’s strategy \textit{Always } R \textit{ is not time-consistent.}

- In some circumstances, Miguel would not follow \textit{Always } R \textit{ during the game,}...

- …and Vanesa knows he won’t.

- So, if Vanesa moves first, she will choose \textit{F}, even if Miguel \textit{says} he will follow \textit{Always } R \textit{.}

- To show this, we need a different model of the game: the \textit{extensive-form game}. 
Extensive-Form Games

- Extensive-form games are described with a *game tree*.
  - The first time period is at the top of the tree.
  - Each level of the tree designates a time period and the player who has a turn to move in that time period.
- Each *branch* of the tree describes an *action* the player can choose.
- Each *node* (where branches meet) describes what a player knows before she moves.
- A *strategy* is a complete plan that states what action a player should take at every one of her nodes.
- Each player’s payoffs are given at the bottom of the tree.

Comparison: Normal Form vs. Extensive Form

- Can you see the connection between the two forms?

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<tr>
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<th>Always F</th>
<th>Miguel Opposite</th>
<th>Always R</th>
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<td>Vanessa</td>
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<td>F</td>
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<th></th>
<th>F</th>
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<tbody>
<tr>
<td>Miguel</td>
<td>F</td>
<td>R</td>
<td></td>
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</tr>
<tr>
<td>F</td>
<td>(2,1)</td>
<td>(0,0)</td>
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<td>R</td>
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<td>t = 2</td>
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Battle of the Sexes in Extensive Form

- Vanesa moves first.
  - She has beliefs but no information.
  - She can choose football…
  - …or opera
- Then it is Miguel’s turn.
  - He looks at Vanesa’s ticket.
    - He sees football…
    - …or opera.
  - If he sees football,
    - He can choose football…
    - …or opera.
    - If he chooses football, Vanesa gets 2 and he gets 1.
    - If he chooses opera, Vanesa gets 0 and he gets 0.
  - If he sees opera,
    - He can choose football…
    - …or opera.
    - If he chooses football, Vanesa gets 0 and he gets 0.
    - If he chooses opera, Vanesa gets 1 and he gets 2.
Subgame-Perfect Equilibrium

- In a subgame-perfect equilibrium, all strategies are time-consistent, ...
- ...that is, no one wants to change his strategy during the game.
- We break the game into subgames.
- Miguel has two subgames:
  - \langle F \rangle and \langle R \rangle
  - Each of Miguel’s subgames corresponds to the game he faces after he finds out what Vanesa did.
- Vanesa has one subgame \langle \rangle, the whole thing, because she cannot move after she finds out what Miguel did.
- An equilibrium is subgame-perfect, if and only if it creates a Nash equilibrium in every subgame.

Finding the Subgame-Perfect Equilibrium

- To find a subgame-perfect equilibrium, we work backwards from the last time period.
- This method is called backwards induction.
- What is Miguel’s best response in subgame \langle F \rangle?
  - He would choose \( F \) and get 1.
  - \( F \) is the Nash equilibrium of subgame \langle F \rangle (because \( F \) is his best response).
- What is Miguel’s best response in subgame \langle R \rangle?
  - He would choose \( R \) and get 2.
  - \( R \) is the Nash equilibrium of subgame \langle R \rangle.
- If we look at the two subgames together, we can see Miguel’s equilibrium strategy (complete plan).
- His equilibrium strategy is Copy. Why?
Vanesa can predict that if Miguel is rational, his strategy must be \textit{Copy}.

So, what is Vanesa’s best response in her subgame \{\}\?  
- If she chooses \textbf{F}, Miguel will choose \textbf{F}, and she will get 2.
- But if she chooses \textbf{R}, Miguel will choose \textbf{R}, and she will get 1.

So Vanesa’s Nash equilibrium strategy is \textbf{F}.

\[ \langle \textbf{F}, \textit{Copy} \rangle \] is a unique subgame-perfect [time-consistent] equilibrium.  
- \[ \langle \textbf{F}, \textit{Copy} \rangle \] creates a Nash equilibrium in every subgame.
- In \[ \langle \textbf{F}, \textit{Copy} \rangle \], Vanesa gets 2; Miguel gets 1.

Note that \[ \langle \textbf{R}, \textit{Always R} \rangle \] is NOT a subgame-perfect equilibrium,…

…because in Miguel’s subgame \[ \langle \textbf{F} \rangle \], the strategy \textbf{R} is not a best response or a Nash equilibrium strategy.

If Vanesa had chosen \textbf{F}, Miguel would not choose \textbf{R}.

\textit{Always R} is NOT time-consistent.
Vanesa’s Advantage

In the subgame-perfect equilibrium, Vanesa moves first. She has no information.

When Miguel moves, he already knows what Vanesa had done.

Miguel has the information advantage, yet Vanesa gets 2 and Miguel gets only 1. Why?

Because Vanesa makes a commitment before Miguel gets to move.

In business and in life, commitment is a big advantage.

But in other settings, information may prove to give a bigger advantage.

Clicker Question
**Example: Pedestrian Crossing**

- You are crossing the road. You can choose to either *wait for cars to pass (W)* or *cross the road without waiting (C)*.

- The driver on the road can either *stop for pedestrians (S)* or *keep going (G)*.

- You prefer \(\langle C, S \rangle\), but the driver prefers \(\langle W, G \rangle\).

- \(\langle C, G \rangle\) has terrible negative payoffs for you and the driver (you are dead, and the driver is in prison).

- If you move first, you can commit to crossing the road \(C\), force the driver to stop \(S\), and obtain your preferred result \(\langle C, S \rangle\).

- What would you do in real life 😊?

*As an exercise, I suggest that you draw the game tree, insert reasonable payoffs, and find the subgame perfect equilibrium.*

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**Matching Pennies: Static Version**

- Remember “Matching Pennies”, the offense vs. defense game?

- *Eva* and *Esther* simultaneously put a penny on the table. *(Each *chooses* heads or tails—they don’t flip the coin.)*

- If *Esther* matches *Eva* (both heads or both tails), then *Eva* pays *Esther* $1.

- But if *Esther* fails to match *Eva* (one is heads, one is tails) *Esther* pays *Eva* $1.

- The game has no Nash equilibrium with pure (nonrandom) strategies.
Matching Pennies: Dynamic Version

- Now suppose that **Eva** moves first.
- **Esther** sees **Eva**’s move, then she moves.
- **Esther** wants to match **Eva**’s move.
- Which player has the advantage, **Eva** or **Esther**?
- We analyze the extensive-form game.

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Matching Pennies in Extensive Form

- What does **Esther** do in her subgames?
  - Esther uses strategy **Copy**.

- What does Eva do in her subgame?
  - If **Eva** chooses **H**, she gets **−1**.
  - If **Eva** chooses **T**, she gets **−1**.
  - Both **H** and **T** are best responses (although both are bad).

- Two subgame-perfect equilibria: \( <H, \text{Copy}> \) and \( <T, \text{Copy}> \)
In both subgame-perfect equilibria, *Eva*, who moves first, gets $-1$, …

and *Esther*, who moves second, gets $+1$.

Even though *Eva* has the power of *commitment*, she loses,…

and *Esther*, who has more *information*, wins.

In games of offense versus defense, information seems more important than commitment.

**Example:** Microsoft waits for another company to build a software application and uses its idea.

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**Dynamic Cournot Duoply**  
(Stackelberg Competition)

Remember the static game between *L’Eau* and *N’Eau*?

- Demand curve was $Q_D = 120 - P$.
- Cost was given by $AC \equiv MC \equiv 0$.
- *L’Eau* sets $q_L$ and *N’Eau* sets $q_N$ at the same time.
- *L’Eau*’s best response to $q_N$ is $q_L^* = \frac{1}{2}(120 - q_N)$,…
- …and *N’Eau*’s is $q_N^* = \frac{1}{2}(120 - q_L)$.
- Equilibrium: $q_L^* = 40$, $q_N^* = 40$, $P = 40$.
- Profits: $Y_L = Y_N = 1600$, CS = 3200. Why?
Now suppose \( L’Eau \) sets \( q_L \) first.

\( N’Eau \) sees \( q_L \), and then he sets \( q_N \) based on the value of \( q_L \).

What will happen? Will the results change?

\( q_L^* = 60, \; q_N^* = 30, \; Y_L = 1800, \; Y_N = 900, \; CS = 4050. \)

- Can you derive these results? \([NOT \ required \ for \ exam]\)

\( L’Eau \), the first firm, will have greater profits than \( N’Eau \),…

…because, in this game, commitment is more important than information is.
Questions

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