

Lecture 23. Offense vs. Defense & Dynamic Games

Final Exam Schedule

EC101 *EE*, Saturday 12/16, 3:00-5:00, Law Auditorium

EC101 *DD*, Tuesday 12/19, 12:30-2:30, STO B50

(DD Students with exam conflicts who completed
the Doodle Survey will take the exam on Saturday.)



Clicker Question

In the long-run equilibrium of monopolistic
competition,

Using Game Theory to Analyze *Offense versus Defense*

- In many competitive situations the **offense** of one competitor battles the **defense** of the other.
 - True of many sports like soccer, basketball and American football.

- If the defense matches the offense, then the defense wins.

- If not, the offense wins.

■ **Example:** Military Strategies

- Attack from the **Left** or from the **Right**
- If the enemy correctly anticipates your action, you lose the battle.

■ **Example:** Business Strategies

- Master Card is better off if it anticipates Visa's market-penetration strategy, but...
- Visa would be better off if Master Card guessed wrong.

Matching Pennies

- “*Matching pennies*” is a game-theory model of offense-versus-defense.
- In this example, *Eva* plays offense; *Esther* plays defense.
- *Eva* and *Esther* each puts a penny on the table at the same time.
- If *Esther* matches *Eva* (both heads or both tails), then *Eva* pays *Esther* \$1.
- But if *Esther* fails to match *Eva* (one heads, one tails) *Esther* pays *Eva* \$1
- This is called a “*zero-sum game*,” because whatever amount one player wins, the other must lose.
- The game has no Nash equilibrium with pure strategies (non-randomized actions).

| | | <i>Esther</i> | |
|------------|----------|---------------|----------|
| | | <i>H</i> | <i>T</i> |
| <i>Eva</i> | <i>H</i> | 1 | -1 |
| | <i>T</i> | -1 | 1 |

Clicker Question

In Matching Pennies,

Dynamic Games

- So far, we've analyzed **static games**, in which all players move at the same time.
- Now we will examine **dynamic games**, in which players move at different times, possibly with different information.
- **Dynamic Game Example:** Airline fares
 - British Airways sets its Boston-London fares.
 - Then, Delta Airlines sets its Boston-London fares.

The Battle of the Sexes: Static Version

- Remember the **Battle of the Sexes**?
- Vanesa wants to go to a football match **F**, but Miguel wants to go to the opera **R**.
- If they both do **F**, then Vanesa gets utility **2**, and Miguel gets **1**,
- and if they both do **R**, then Vanesa gets **1** and Miguel gets **2**.
- But if they do different things, then both get **0**.
- Both must choose their strategies **at the same time**, without knowing what the other has done.
- There are two Nash equilibria: $\langle F, F \rangle$ and $\langle R, R \rangle$.

| | | <i>Miguel</i> | |
|---------------|----------|---------------|----------|
| | | F | R |
| <i>Vanesa</i> | F | ★ 1 2 | 0 |
| | R | 0 | ★ 2 1 |

The Battle of the Sexes: Dynamic Version

- Now suppose that the players move at different times, first one, then the other.

- For example, suppose that **Vanesa moves first**: she buys a ticket for **either** the football match **or** the opera.

| | | Miguel | |
|--------|---|--------|--------|
| | | F | R |
| Vanesa | F | 1 2 | 0 0 |
| | R | 0 0 | 2 1 |

- She shows Miguel her ticket, so **he knows what she has done**.
- Then Miguel moves**: he buys his ticket for **either** the football match **or** the opera.

- What would happen in this game?

- The answer is clear!

- Vanesa (the selfish beast 😊) will choose football **F**...

- and “force” Miguel to choose football **F** as well.

| | | Miguel | |
|--------|---|--------|--------|
| | | F | R |
| Vanesa | F | 1 2 | 0 0 |
| | R | 0 0 | 2 1 |

- $\langle F, F \rangle$ still looks like a Nash equilibrium.

- We know they won't choose $\langle R, R \rangle$, but is $\langle R, R \rangle$ still an equilibrium?

- To find out, we must model strategies properly.

- If Vanesa moves first, and Miguel sees the result before he moves,...
- ...then the matrix above does not correctly represent the game.

Dynamic-Game Strategies

- A **strategy** is a **complete plan** of action that specifies what a player will do **in every circumstance** that she can observe.

- From what strategies does Vanesa choose?

- **F** and **R** (as before).

- What about Miguel? What are his strategy choices?

- **F** and **R** are **NOT** strategies for Miguel.
- A strategy is a complete plan that might tell you to do different things in each situation you know about.
- Miguel knows whether Vanesa has bought **F** or bought **R**.
- So his strategy must reflect his knowledge of her action.

| | | <i>Miguel</i> | |
|---------------|------------|---------------|--------------|
| | | X | X |
| <i>Vanesa</i> | → F | 1 | 0 |
| | → R | 0 | 2 |

- Miguel's possible strategy choices are the following (with my own nicknames):

- **Always F**: If she bought **F**, I will choose **F**.
If she bought **R**, I will choose **F**.
- **Copy**: If she bought **F**, I will choose **F**.
If she bought **R**, I will choose **R**.
- **Opposite**: If she bought **F**, I will choose **R**.
If she bought **R**, I will choose **F**.
- **Always R**: If she bought **F**, I will choose **R**.
If she bought **R**, I will choose **R**.

- These four strategies form Miguel's strategy space.

Representing the Dynamic Game

- The dynamic Battle of the Sexes can be represented as follows:

| | | <i>Miguel</i> | | | |
|---------------|----------|-----------------|-------------|-----------------|-----------------|
| | | <i>Always F</i> | <i>Copy</i> | <i>Opposite</i> | <i>Always R</i> |
| <i>Vanesa</i> | <i>F</i> | 1 2 | 1 2 | 0 0 | 0 0 |
| | <i>R</i> | 0 0 | 2 1 | 0 0 | 2 1 |

- Notice that if *Vanesa* does *F*, then *Miguel's* strategies *Always F* and *Copy* require the same **actions** and lead to the same payoffs.

- But what are the Nash equilibria of this game?
- If we check each cell, we can see that there are exactly 3 pure-strategy equilibria:

- $\langle F, \text{Always } F \rangle$
- $\langle F, \text{Copy} \rangle$
- $\langle R, \text{Always } R \rangle$

| | | <i>Miguel</i> | | | |
|---------------|----------|-----------------|-------------|-----------------|-----------------|
| | | <i>Always F</i> | <i>Copy</i> | <i>Opposite</i> | <i>Always R</i> |
| <i>Vanesa</i> | <i>F</i> | ★ 1 2 | ★ 1 2 | 0 0 | 0 0 |
| | <i>R</i> | 0 0 | 2 1 | 0 0 | ★ 2 1 |

- In each equilibrium, the players have no incentive at the beginning of the game to deviate from their chosen strategies.
- However, it turns out that **only** $\langle F, \text{Copy} \rangle$ is formed from strategies (plans) that would actually be followed during the game.
- What's wrong with the strategies in the other equilibria?
- **Answer:** They are not *time-consistent* ...

Course Evaluations

Now we'll do the course evaluations.

The lecture will continue afterwards.

The entire evaluation (except for Q4) is about M. Manove. **TF evaluations should be completed in your discussion sections.**

Q5: Substitute for the original question:
"I found the clicker questions useful."

Online Course Evaluations

- Type the following URL into your internet browser's search field: bu.campuslabs.com/courseeval
- Enter your BU login name and Kerberos Password. Complete the evaluation for *this course*.
- Your evaluations are **anonymous**. Instructors will not receive results until final grades have been submitted.
- Comments in the text fields are valued and encouraged. Please try to answer all questions, but if a question is not applicable to you, or if you do not wish to answer it, then you can skip it.
- When you are done, please close your browser.
- If you cannot complete your evaluation in class, please complete it sometime today.

Clicker Question

What is true about
 $\langle R, \text{Copy} \rangle$?

| | | Miguel | | | |
|---------|---|-------------|--------|---------------|-------------|
| | | Always F | Copy | Oppo- site | Always R |
| Vanessa | F | 1 2 | 1 2 | 0 0 | 0 0 |
| | R | 0 0 | 2 1 | 0 0 | 2 1 |

Time Consistency

- A **strategy** is a plan of action that specifies what a player will do **in every circumstance** that she can observe.
- Think of a strategy as a plan made at the beginning of the game.
- The strategy is **time-consistent** if the player is willing to follow her plan as the game progresses no matter what happens.
 - **Example:** Your strategy is to study economics tonight even if your roommate is having a party,...
 - but when the party begins, you succumb to temptation and decide not to study.
 - Your strategy was not time-consistent.

- In our Battle-Sexes example, Vanesa buys her ticket first.
- But if Miguel says he will go to opera no matter what Vanesa does,
- ...wouldn't Vanesa be "forced" to buy an opera ticket?

- It's true that $\langle R, \text{Always } R \rangle$ is a Nash equilibrium!

- But maybe Vanesa would ignore Miguel's statement!

- Vanesa suspects that if she chooses F , Miguel will change his mind about **Always R**.

| | | <i>Miguel</i> | | | |
|---------------|----------|-----------------|-------------|-----------------|-----------------|
| | | <i>Always F</i> | <i>Copy</i> | <i>Opposite</i> | <i>Always R</i> |
| <i>Vanesa</i> | <i>F</i> | 1 | 1 | 0 | 0 |
| | <i>R</i> | 0 | 2 | 0 | ★ 2 |
| | | 2 | 1 | 0 | 1 |

- She thinks: "Miguel might choose **Always R** when he's planning his strategy at the beginning of the game,..."
- "but if I have chosen F , when it's his turn to buy a ticket, Miguel won't follow the **Always-R** plan."
- **Always R** may be an "idle threat" (that Miguel will not carry out), a threat that Vanesa doesn't believe. Maybe it's not time-consistent.

A New Kind of Equilibrium

- In general, the Nash equilibrium does not guarantee that equilibrium strategies will be time consistent,...
- ...because the Nash-equilibrium concept doesn't eliminate idle threats.
- However, there's a special kind of Nash equilibrium that does guarantee time-consistent equilibrium strategies...
- The **subgame-perfect Nash equilibrium**.

Normal-Form and Extensive-Form Games

- So far, we've described games with a matrix in which each row or column represents a player's strategy: the ***normal-form game***.
- But to find a ***subgame-perfect Nash equilibrium*** we need a different game structure: the ***extensive-form game***.
- We'll explain the extensive-form game in the next lecture,...
- ...and we'll use it to find an equilibrium with time-consistent strategies.

Clicker Question

A player's strategy is time consistent if

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