Lecture 23.
Offense vs. Defense & Dynamic Games

Clicker Question

Using Game Theory to Analyze *Offense versus Defense*

- In many competitive situations the ***offense*** of one competitor battles the ***defense*** of the other.
- If the defense matches the offense, then the defense wins.
- If not, the offense wins.

- **Example:** Military Strategies
- **Example:** Business Strategies

Matching Pennies

		Esther	
		H	T
Eva	H	1 −1	−1 1
	T	−1 1	1 −1

- *"Matching pennies"* is a game-theory model of offense-versus-defense.
- In this example, ***Eva*** plays offense; ***Esther*** plays defense.
- ***Eva*** and ***Esther*** each puts a penny on the table at the same time.
- If ***Esther*** matches ***Eva*** (both heads or both tails), then ***Eva*** pays ***Esther*** $1.
- But if ***Esther*** fails to match ***Eva*** (one heads, one tails) ***Esther*** pays ***Eva*** $1
- This is called a "***zero-sum game***," because whatever amount one player wins, the other must lose.
- The game has no Nash equilibrium with pure (non randomized) strategies.

Clicker Question

Dynamic Games

- So far, we've analyzed ***static games***, in which all players move at the same time.
- Now we will examine ***dynamic games***, in which players move at different times.
- **Dynamic Game Example:** Airline fares
  - British Airways sets its Boston-London fares.
  - Then, Delta Airlines sets its Boston-London fares.

The Battle of the Sexes: Static Version

- Remember the ***Battle of the Sexes***?
- Vanesa wants to go to a football match ***F***, but Miguel wants to go to the opera ***R***.
- If they both do ***F***, then Vanesa gets utility ***2*** , and Miguel gets ***1***,
- and if they both do ***R***, then Vanesa gets ***1*** and Miguel gets ***2***.
- But if they do different things, then both get **0**.
- Both must choose their strategies ***at the same time***, without knowing what the other has done.
- There are two Nash equilibria: ⟨***F, F*** ⟩ and ⟨***R, R***⟩.

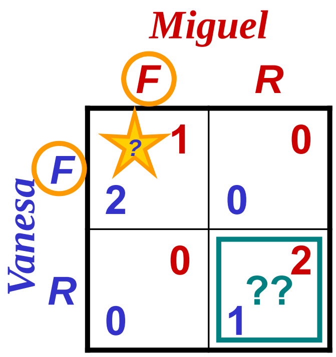
		Miguel	
		F	***R***
Vanesa	***F***	★ 1 2	0 0
	R	0 0	★ 2 1

The Battle of the Sexes: Dynamic Version

- Now suppose that the players move at different times, first one, then the other.
- For example, suppose that ***Vanesa moves first***: she buys a ticket for ***either*** the football match ***or*** the opera.

		Miguel	
		F	R
Vanesa	F	1 2	0 0
	R	0 0	2 1

- She shows Miguel her ticket, so ***he knows what she has done***.
- ***Then Miguel moves:*** he buys his ticket for ***either*** the football match ***or*** the opera.



		Miguel	
		F	R
Vanesa	F	1 2	0 0
	R	0 0	2 1

- What would happen in this game?
- The answer is clear!
- Vanesa (the selfish beast ☺) will choose football ***F***…
- and “force” Miguel to choose football ***F*** as well.
- ***⟨F, F⟩*** still looks like a Nash equilibrium.
- We know they won’t choose ***⟨R, R⟩***, but is ***⟨R, R⟩*** still an equilibrium?
- To find out, we must model strategies properly.
- If Vanesa moves first, and Miguel see the result before he moves,…
- …then the matrix above does not correctly represent the game.

Dynamic-Game Strategies

- A ***strategy*** is a ***complete plan*** of action that specifies what a player will do ***in every circumstance*** that she can observe.
- From what strategies does Vanesa choose?
  - ***F*** and ***R*** (as before).

		Miguel	
		~~***F***~~	~~***R***~~
Vanesa	***F***	**1** **2**	**0** **0**
	R	**0** **0**	**2** **1**

- What about Miguel? What are his strategy choices?
  - ***F*** and ***R*** are **NOT** strategies for Miguel.
  - A strategy is a plan that might tell you to do different things in each situation you know about.
  - Miguel knows Vanesa has bought ***F*** or bought ***R***.
  - So his strategy must reflect his knowledge of her action.

- Miguel's possible strategy choices are the following (with my own nicknames):
  - ***Always F*** : If she bought ***F***, I will choose ***F*** .
    If she bought ***R***, I will choose ***F*** .
  - ***Copy*** : If she bought ***F***, I will choose ***F*** .
    If she bought ***R***, I will choose ***R*** .
  - ***Opposite*** : If she bought ***F***, I will choose ***R*** .
    If she bought ***R***, I will choose ***F*** .
  - ***Always R*** : If she bought ***F***, I will choose ***R*** .
    If she bought ***R***, I will choose ***R*** .
- These four strategies form Miguel's strategy space.

Representing the Dynamic Game

- The dynamic Battle of the Sexes can be represented as follows:

Miguel

		Always F	*Copy*	*Oppo-site*	*Always R*
Vanesa	*F*	1 2	1 2	0 0	0 0
	R	0 0	2 1	0 0	2 1

- Notice that if ***Vanesa*** does ***F***, then ***Miguel***'s strategies ***Always F*** and ***Copy*** require the same actions and lead to the same payoffs.

- But what are the Nash equilibria of this game?
- If we check each cell, we can see that there are exactly 3 pure-strategy equilibria:
  - ⟨***F***, ***Always F*** ⟩
  - ⟨***F***, ***Copy*** ⟩
  - ⟨***R***, ***Always R*** ⟩

Miguel

		Always F	*Copy*	*Oppo-site*	*Always R*
Vanesa	*F*	★ 1 2	★ 1 2	0 0	0 0
	R	0 0	2 1	0 0	★ 2 1

- In each equilibrium, the players have no incentive at the beginning of the game to deviate from their chosen strategies.
- However, it turns out that **only** ⟨***F***, ***Copy*** ⟩ is formed from strategies (plans) that would actually be followed during the game.
- What's wrong with the strategies in the other equilibria?
- **Answer:** They are not ***time-consistent*** …

Course Evaluations

Now we'll do the course evaluations.

The lecture will continue afterwards.

The entire evaluation (except for Q4) is about M. Manove. **The TFs will distribute their own evaluations.**

Q5: Substitute for the original question:
"I found the clicker questions useful."

Clicker Question

Time Consistency

- A ***strategy*** is a plan of action that specifies what a player will do ***in every circumstance*** that she can observe.
- Think of a strategy as a plan made at the beginning of the game.
- The strategy is ***time-consistent*** if the player is willing to follow her plan as the game progresses.
  - **Example:** Your strategy is to study economics tonight even if your roommate is having a party,…
  - but when the party begins, you succumb to temptation and decide not to study.
  - Your strategy was not time-consistent.

- In our Battle-Sexes example, Vanesa buys her ticket first.
- But if Miguel says he will go to opera no matter what Vanesa does,
- wouldn't Vanesa be "forced" to buy an opera ticket?
  - ⟨***R***, ***Always R*** ⟩ is a Nash equilibrium!
- Maybe Vanesa would ignore Miguel's statement!
  - Vanesa suspects that if she chooses ***F***, Miguel will change his mind about ***Always R***.
  - She thinks: Miguel might choose ***Always R*** when he's planning his strategy at the beginning of the game,…
  - but **when it's his turn** to buy a ticket, Miguel may be unwilling to follow the ***Always-R*** plan if I have chosen ***F***.
  - ***Always R*** may be an "idle threat" (that Miguel will not carry out), a threat that Vanesa doesn't believe.

		Miguel			
		Always F	***Copy***	***Oppo-site***	***Always R***
Vanesa	***F***	1 2	1 2	0 0	0 0
	R	0 0	2 1	0 0	★ 2 1 ??

A New Kind of Equilibrium

- In general, the Nash equilibrium does not guarantee that equilibrium strategies will be time consistent,…
- …because the Nash-equilibrium concept doesn't eliminate idle threats.
- However, there's a special kind of Nash equilibrium that does guarantee time-consistent equilibrium strategies:
  the ***subgame-perfect Nash equilibrium***.

Normal-Form and Extensive-Form Games

- So far, we've described games with a matrix in which each row or column represents a player's strategy: the ***normal-form game***.
- But to find a ***subgame-perfect Nash equilibrium*** we need a different game structure:
  the ***extensive-form game***.
- We'll explain the extensive-form game in the next lecture,…
- …and we'll use it to find an equilibrium with time-consistent strategies.

Clicker Question

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