Abstraction and Control for Swarms of Robots

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Abstract. This paper addresses the design of simple robot behaviors that realize emergent group behaviors. We present a method to coordinate a large number of under-actuated robots by designing control laws on a small dimensional *abstraction* manifold, independent of the number and ordering of the robots. The abstraction manifold has a product structure consisting of elements of a Lie group that capture the position and orientation of the ensemble in the world frame, and elements of a shape manifold that provide an intrinsic description of the distribution of team members relative to one another. We design decoupled controls for regulating the group and the shape variables. The realization of the controller on each robot requires the feedback of the robot state, and the state on the abstraction manifold. We present experimental results with a team of five car-like robots equipped with omnidirectional cameras and IEEE 802.11b networking.

1 Introduction

There are many examples of swarms in nature where simple, local behaviors, lead to a wide array of complex, group behaviors [1,2]. Behavior-based approaches in robotics have used similar ideas to demonstrate cooperative, multi-robot behaviors [3]. However, the inverse problem of designing robot behaviors to achieve a desired group behavior has proved to be very difficult.

There are some inverse problems that have been solved. For example, if the robots are required to form a rigid formation or a virtual structure, the team can be viewed as a left invariant control system on $SE(l)$ ($l = 2, 3$), and the individual trajectories are $SE(l)$ - orbits [4]. The literature on stabilization and control of rigid formations is rather extensive. The interaction between the robots involves leader-follower controllers [5] and can be described by *formation graphs* whose edges represent inter-robot constraints that must be satisfied. Characterizations of rigid formations can be found in [6,4], and variants on the robot controllers leading to different forms of inter-robot interactions can be found in [7–9].

While a rigid formation may be appropriate for applications such as cooperative manipulation [10], formation flying [11], and cooperative sensing [12], such an approach is, in general, too restrictive for environments with obstacles. In addition, a rigid formation generally requires robots to be identified and formations to be pre-planned [5].

P. Dario and R. Chatila (Eds.): Robotics Research, STAR 15, pp. 224–233, 2005.

We introduce a general framework for the inverse problem of designing robot controls that can realize desired collective behaviors. We primarily focus on the synthesis of emergent spatial patterns at the group level as a result of designed interactions at the level of individual robots. Our specific goal is to control a large number of car-like robots that may be tasked to move while maintaining a desired spatial distribution. An example of simple task may be to move hundreds of robots from arbitrary initial configurations through a obstacle-filled environment while staying grouped together. The large number makes the synthesis of motion plans for each individual robot intractable. Our framework involves the abstraction of the collection of robots at the group level so that the motion planning for the group can be done on a lower-dimensional manifold, whose dimension is independent of the number of robots or the labelling of these robots. The control problem then is to design individual robot controllers so that the motion plan developed at the abstract higher level can be realized, and to ensure that the sensing and control tasks and the related computations that must be performed by individual robots do not scale poorly as the number of robots is increased.

The key contributions of this work are as follows. First, we define our abstraction manifold to have a product structure of a Lie group, which captures the dependence of the ensemble on the world frame, and a shape manifold, which is an intrinsic description of the team. We show how decoupled controllers can be designed for the group and shape variables. The realization of the controller on each robot requires the feedback of the robot state, and the state on the abstraction manifold. We discuss the design and implementation of an observer that can estimate the state of the group and broadcast the estimate to all robots. Finally, we present experimental results with a team of five car-like robots equipped with omnidirectional cameras and IEEE 802.11b networking using this architecture and show how simple robot controllers can be used to realize group behaviors in a principled manner.

2 Problem Formulation

Consider N identical Hilare-like planar robots, each described by a 3-dimensional state vector $x^i = [x_1^i, x_2^i, x_3^i]^T$, $i = 1, \dots N$, where (x_1^i, x_2^i) give the Cartesian coordinates of the center of the drive axle and x_3^i measures the orientation of the robot frame in a given world frame $\{W\}$. Each robot is modeled as a kinematic, drift free control system

$$\dot{x}^i = G(x^i)u^i = g_1(x^i)u_1^i + g_2(x^i)u_2^i \tag{1}$$

where the control vector fields are given by

$$g_1(x) = [\cos x_3,\ \sin x_3,\ 0]^T\ ,\ \ g_2(x) = [0,\ 0,\ 1]^T$$

for $x = [x_1, x_2, x_3]^T$. The control $u^i = (u_1^i, u_2^i)$ consists of driving and steering speeds. On each robot we pick a reference point P_i along the longitudinal axis of symmetry perpendicular to the drive axle but different from the robot center. The

Cartesian coordinates of the reference points, $q^i = (q_1^i, q_2^i)$, are used to formulate cooperative tasks. In other words, for each robot i, $i = 1, \ldots, N$, we define an output map

$$q^i = h(x^i) \tag{2}$$

where (omitting the superscript i) h is given by

$$h(x) = [x_1 + d\cos x_3,\ x_2 + d\sin x_3]^T \tag{3}$$

Note that the choice of output function h together with the linearity of dynamics (1) in u^i leads to a linear nonsingular relationship between the derivative of the output $\dot{q}^i$ and the control variables $\dot{q}^i = dh(x^i)G(x^i)u^i$, unless $d = 0$.

Problem 1 (Control). Design control laws u^i, $i = 1, \ldots, N$ so that the team of robots accomplishes a cooperative task formulated in terms of the reference points q^i.

We first define new inputs

$$\dot{q}^i = v^i \tag{4}$$

which are related to the original ones by

$$u^i = A(x^i)v^i, \tag{5}$$

$$A(x) = (dh(x)G(x))^{-1} = \begin{bmatrix} \cos x_3 & \sin x_3 \\ -\frac{\sin x_3}{d} & \frac{\cos x_3}{d} \end{bmatrix} \tag{6}$$

where dh denotes the differential of h. Equations (1), (2), (4) and (5) represent an input-output feedback linearization problem [13]. The next natural step would be to set $v^i = \dot{q}^{id} + k(q^{id} - q^i)$, $k > 0$ so that q^i exponentially tracks a given desired trajectory $q^{id}(t)$. However, this would require us to obtain specifications of desired trajectories for each individual robot, something that is not practical for large numbers of robots. Instead, we will show how the redefined inputs v^i can be designed so that the robots described by the reference points q^i, $i = 1, \ldots, N$ have a desired collective behavior.

3 Abstraction

Consider the $2N$-dimensional system describing the team of robots:

$$\dot{q} = v,\ q \in Q,\ v \in V \tag{7}$$

$$\begin{aligned} Q &= \{q| = (q^1, \ldots, q^N) \in \mathbb{R}^{2N}\}, \\ V &= \{v| = (v^1, \ldots, v^N) \in \mathbb{R}^{2N}\}, \end{aligned} \tag{8}$$

with the canonical projection

$$\pi_i(q) = q^i,\ \pi_i(v) = v^i,\ i = 1, \ldots, N \tag{9}$$

We define an abstraction of the team based as a map

$$\phi : Q \to A,\ \phi(q) = a \tag{10}$$

which is required to satisfy the following properties:

(i) The map ϕ is a surjective submersion.
(ii) The map ϕ is invariant to permutations of the robots and the dimension n of A is not dependent on the number of robots N.
(iii) We require that A have a product structure

$$A = G \times S,\ a = (g,\, s),\ \phi = (\phi_g,\, \phi_s) \tag{11}$$

where G is the Euclidean motion Lie group.
(iv) The map ϕ is left-invariant. In other words, if $\phi(q) = (g, s)$ and $\bar{g} \in G$ is a generic element, $\phi(\bar{g}q) = (\bar{g}g, s)$.
(v) The control systems on the group G and shape S are decoupled.

The key idea underlying the abstraction is simple. Instead of designing team motions in the high-dimensional space Q, we want to be able to plan collective behaviors on the lower-dimensional manifold A. The submersion condition (i) on ϕ implies the surjectivity of the differential $d\phi$ at any $q \in Q$. This allows us to think of vector fields (behaviors) on TQ and corresponding abstract behaviors on TA via the push-forward map $w = d\phi(q)v$. Thus, we can derive control inputs v that will realize any vector field (behavior) w on the abstract space as follows:

$$v = d\phi^T (d\phi d\phi^T)^{-1} w \tag{12}$$

where $w = \dot{a}$ are new inputs on the abstraction manifold. It is not difficult to see that (12) guarantees that the energy spent by the vehicles to produce a specified abstract behavior $X_A \in TA$ is minimized [14]. Further, the abstract state a is at rest ($\dot{a} = w = 0$) if and only if all the robots (q^i) are at rest ($\dot{q} = v = 0$). This guarantees that each individual motion can be "seen" in the small dimensional manifold A.

Requirement (ii) ensures the dimension of the control problem is independent of the number of agents, and the controllers are robust to individual failures. The main idea of requirement (iii) is to have a decomposition in the abstract manifold, $a = (g, s)$. The element $g \in G$ is called the *group* element and describes the *pose* of the team and the element $s \in S$ describes the *shape* of the team. Since we only address planar robots in this paper, G is $SE(2)$. The left-invariance property (iv) will guarantee that the shape is an intrinsic property of the formation and is unaffected by the motion of the team. The decoupling property in requirement (v) means that controllers for regulating the shape of the formation can be designed independently from controllers that regulate the motion of the team. Specifically, the control systems on G and S are decoupled if $d\phi_g$ and $d\phi_s$ are orthogonal as subspaces (we assume that Q is equiped with an Euclidean metric, although we could consider the more general case of a Riemannian metric).

Finally, it is desirable to limit the inter-robot communication in the overall control scheme. We propose an architecture where the control law u^i of a robot only depends on its own state x^i and the state of the low dimensional abstraction manifold $a \in A$. Since $u^i = A(x^i)v^i$ and $q^i = h(x^i)$, this is achieved if $v^i = \pi_i(v) = v^i(q^i, a)$.

4 Control

In this section we define a physically significant abstraction (10) with a product structure (11) as follows. For an arbitrary configuration $q \in Q$, the group part g of the abstract state a is defined by $g = (R, \mu) \in G = SE(2)$. Let

$$\mu = \frac{1}{N} \sum_{i=1}^{N} q_i \in \mathbb{R}^2 \tag{13}$$

Let the rotation $R \in SO(2)$ be parameterized by $\theta \in (-\pi/2, \pi/2)$, Then, by definition,

$$\theta = \frac{1}{2} \operatorname{atan2} \left(\sum_{i=1}^{N} (q_i - \mu)^T E_1 (q_i - \mu), \sum_{i=1}^{N} (q_i - \mu)^T E_2 (q_i - \mu) \right) \tag{14}$$

Let the shape variable be a two-dimensional vector $s = [s_1,\ s_2]$ defined by

$$\begin{aligned} s_1 &= \tfrac{1}{2(N-1)} \textstyle\sum_{i=1}^{N} (q_i - \mu)^T H_1 (q_i - \mu), \\ s_2 &= \tfrac{1}{2(N-1)} \textstyle\sum_{i=1}^{N} (q_i - \mu)^T H_2 (q_i - \mu) \end{aligned} \tag{15}$$

where

$$H_1 = I_2 + R^2 E_2, \quad H_2 = I_2 - R^2 E_2, \quad H_3 = R^2 E_1, \tag{16}$$

$$E_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{17}$$

It is obvious that the abstraction defined by (13), (14), and (15) has the required product structure and is independent of the number and the permutations of the robots (requirements (ii), (iii)). In [14], it is shown that it also satisfies the left invariance property (iv) and $d\mu$, $d\theta$, ds_1, and ds_2 are orthogonal with respect to the Euclidean metric on Q. Therefore, each of the abstract variables can be controlled separately and requirement (v) is satisfied. Then it makes sense to design separate controls $w = (w_\mu, w_\theta, w_{s_1}, w_{s_2})$ at a point $a = (\mu, \theta, s_1, s_2)$.

A group behavior is described by a desired trajectory $(g^d(t), s^d(t))$, which can be tracked using a controller on the abstraction manifold A:

$$\dot{\xi} = w_\xi = k_\xi (\xi^d(t) - \xi(t)) + \dot{\xi}^d(t), \quad \xi \in \{\mu, \theta, s_1, s_2\} \tag{18}$$

where $k_\mu \in \mathbb{R}^{2\times 2}$ is a positive definite matrix and k_θ, $k_{s_{1,2}} > 0$.

4.1 Significance

As shown in [14], there are two slightly different interpretations of the abstraction defined by (13), (14), and (15). Let

$$\Sigma = \frac{1}{N-1} \sum_{i=1}^{N} (q_i - \mu)(q_i - \mu)^T, \quad \Gamma = -(N-1) E_3 \Sigma E_3 \tag{19}$$

It is easy to see that Γ and $(N-1)\Sigma$ have the same eigenstructure.

μ and Γ in (13) and (19) can be seen as the centroid and inertia tensor of the system of particles q^i with respect to $\{W\}$. Let $\{M\}$ define a virtual frame with pose $g = (R, \mu)$ in $\{W\}$. The rotation equation (14) defines the orientation of the virtual frame so that the inertia tensor of the system of points in $\{M\}$ is diagonal. $(N-1)s_1$ and $(N-1)s_2$ are the eigenvalues of the tensor and are therefore measures of the spatial distribution of the reference points q^i along the axis of the virtual frame $\{M\}$. Specifically, in [14] it is shown that *an ensamble of N robots described by a 5 - dimensional abstract variable $a = (g, s) = (R, \mu, s_1, s_2)$ is enclosed in a rectangle centered at μ and rotated by $R \in SO(2)$ in the world frame* $\{W\}$. *The sides of the rectangle are given by $2\sqrt{(N-1)s_1}$ and $2\sqrt{(N-1)s_2}$.*

Alternatively, μ and Σ given by (13) and (19) can be interpreted as sample mean and covariance of a random variable with realizations q^i. If the random variable is normally distributed, then, for a sufficiently large N, μ and Σ converge to the real parameters of the normal distribution. R in (14) is the rotation that diagonalizes the covariance and s_1, s_2 are the eigenvalues of the covariance matrix. This means that, for a large number of normally distributed reference points (q^i), μ, R, s_1 and s_2 give the pose and semiaxes of a *concentration ellipse*:

$$(x-\mu)^T \Sigma^{-1} (x-\mu) = c, \quad c = -2\ln(1-p) \tag{20}$$

The ellipse in (20), also called *equiprobability ellipse*, has the property that p percent of the points are inside it, and can be used as a spanning region for our robots. Therefore we can make the following statement: *p percent of a large number N of normally distributed points described by a 5 - dimensional abstract variable $a = (g, s) = (R, \mu, s_1, s_2)$ is enclosed in an ellipse centered at μ, rotated by $R \in SO(2)$ in the world frame $\{W\}$ and with semiaxes $\sqrt{cs_1}$ and $\sqrt{cs_2}$, where c is given by (20).* Even though the normal distribution assumption might seem very restrictive, we show in [14] that it is enough that the reference points q^i be normally distributed in the initial configuration. Our controls laws preserve the normal distribution.

The abstraction based on the spanning rectangle has the advantage that it provides a rigorous bound for the region occupied by the robots and does not rely on any assumption on the distribution of the robots. The main disadvantage is that this estimate becomes too conservative when the number of robots is large. On the other hand, the size of the concentration ellipse does not scale with the number of robots, which makes this approach very attractive for very large N. However, it has the disadvantage of assuming a normally distributed initial configuration of the team and does not provide a rigurous bound for the region occupied by the robots. Roughly speaking, $(1-p)N$ are left out of the p-ellipse. Increasing p will decrease the number of the robots which might be outside but will also increase the size of the ellipsoid.

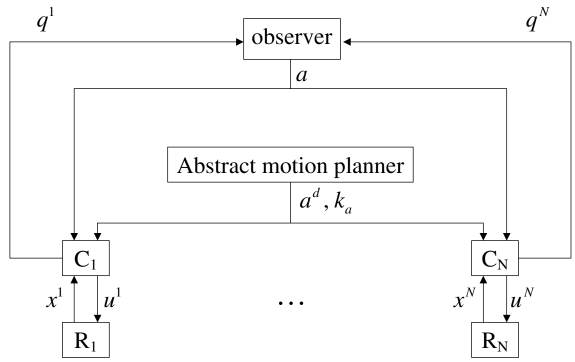

Fig. 1. The control architecture showing decentralized controllers C_i, with a centralized abstract motion planner and an observer that provides an estimate of the abstract state.

4.2 Individual Control Laws

In [14], it is shown that the map ϕ defined by (13), (14), and (15) satisfies the submersion condition (i) if and only if $s_1 \neq 0$ and $s_2 \neq 0$. These cases of zero shape physically correspond to degenerate situations when all the robots are on the Oy and Ox axis of the formation frame $\{M\}$, respectively. Excluding these degenerate cases, $d\phi d\phi^T$ is invertible and the projection (9) of (12) leads to the following velocity for the reference points P_i, $i = 1, \ldots, N$:

$$v^i = w_\mu + \frac{s_1 - s_2}{s_1 + s_2} H_3(q_i - \mu) w_\theta + \frac{1}{4s_1} H_1(q_i - \mu) w_{s_1} + \frac{1}{4s_2} H_2(q_i - \mu) w_{s_2} \quad (21)$$

The solution to Problem 1 is therefore given by u^i defined by (5) and (21).

The control architecture is shown in Figure 1. The motion plan on the abstraction manifold, which consists of the desired group and shape motion, must be made available by a centralized motion planner. Note, that the ith robot, R_i runs the controller C_i given by (5,21), requiring only feedback of the robot state x_i and the abstract state, a (and not states of other robots in the team). As shown in the figure, it is necessary to create an observer that provides partial state feedback in terms of estimates of the low-dimensional abstract state vector.

In [14], it is shown that if control law (21) is applied to all the robots, then the set of points q^i undergoes an affine transformation in the plane. Therefore, control law (21) can be used for formations in which preserving properties like collinearity, ratios of distances on lines, and parallelism is important. Even more interesting, it is known that affine transformations preserve the normal distribution. This means that if the robots are initially normally distributed, by applying the control laws (21), they remain normally distributed. The 5-dimensional abstract state, interpreted as sample mean μ and sample covariance Σ, gives us control over the pose, aspect ratio and size of the concentration ellipse as defined above.

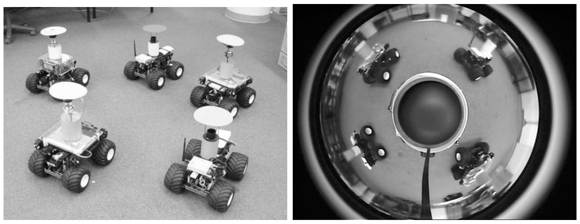

Fig. 2. The GRASP Lab. robots (left) and a sample image from an omnidirectional camera (right).

4.3 Internal Dynamics

Note that (18) only guarantees the desired behavior and therefore the boundness of the 5 - dimensional $a \in A$. The next step is to prove the boundness of the internal dynamics. This usually implies a change of coordinates and explicit calculation of the zero dynamics. Fortunately, the boundness of $a \in A$ together with the definition of ϕ easily imply the boundness of each of q^i, $i = 1, \ldots, N$ [14]. Moreover, in the stabilization to a point case, it can be proven that for any μ^d, θ^d, s_1^d, s_2^d, the closed loop system globally asymptotically converges on Q to the equilibrium manifold $\mu = \mu^d$, $\theta = \theta^d$, $s_1 = s_1^d$, $s_2 = s_2^d$. We conclude that the overall system is well behaved on Q, and therefore, each individual output q^i is well behaved. Moreover, the remaining 1 - dimensional internal dynamics of each robot can also be proved to be bounded [5].

5 Experimental Results

Our experiments were performed using a team of five car-like robots. Each robot is equipped with its own processor and an omnidirectional camera as shown in Figure 2. The low-level controllers, the communication protocols and state estimators are discussed in [15].

In our implementation, an overhead camera mounted on a UAV provides the feedback necessary for estimating the abstract state in Figure 1. However, the results in this paper were obtained in an indoor setting with a fixed, calibrated, overhead camera. A fixed supervisory computer estimates the 5-dimensional team variable $a = (\mu, \theta, s_1, s_2)$ and broadcasts it to the robots in the team. The supervisory computer also broadcasts the abstract motion plan. In the experiments shown here, the team behavior in (18) corresponds to stabilization to a point a^d on the abstraction manifold A.

It is important to note that the control computers on the robots do not communicate with others on the team. Further, the computations at each node are independent of the number of robots. In our implementation, the six computers have different processing speed and therefore, different update rates, the slowest update rate corresponding to approximately $15\,Hz$.

Two sample experimental runs are shown in this paper. Limitations on space preclude us from providing more information, including simulation results for hundreds of robots in an obstacle-cluttered environment [14].

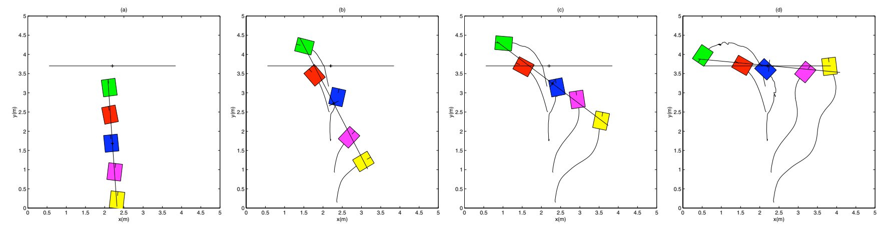

Fig. 3. The position and orientation of the team is stabilized at desired values while shape is preserved.

Fig. 4. Sequence of snapshots in an expansion maneuver - illustrates control of shape while pose in preserved.

In the first experiment we show how the pose of the team can be controlled while shape is preserved, illustrating the decoupling property of controllers (5), (21). The robots are initially "almost" aligned with the Oy axes of the world frame: $\mu = (2.2020, 1.6817)$, $\theta = -1.499\,rad$, $s_1 = 0.5417$, $s_2 = 1.6798e-4$. We use controllers (18) with $k_\mu = 4I_2$, $k_\theta = 4$, $k_{s_1} = k_{s_2} = 0$ without derivative terms to stabilize the team at $\mu^d = (2.2, 3.7)$, $\theta^d = 0$, and the shape, according to our theoretical results, should be preserved. The ground truth information captured from a calibrated, overhead camera are shown in Figure 3.

The second experiment illustrates an expansion maneuver. Instead of plotting the experimental data, we show four snapshots from the actual experiment in Figure 4. The robots were initially grouped in a small circle $s_1 = s_2 = 0.0738$ around $\mu = (2.4607, 2.6185)$. We again used the stabilizing controllers (18) without derivative terms but this time with $k_\mu = 0$, $k_\theta = 0$, $k_{s_1} = k_{s_2} = 4$ to stabilize the team at $s_1^d = s_2^d = 0.6078$. The pose of the team, as predicted by our theoretical results, is preserved.

6 Conclusion

We propose a framework and control algorithms for coordinating a swarm of robots that allows the synthesis of a class of emergent group behaviors in a scalable manner. The first key idea is the abstraction of the team to a low-dimensional abstract state vector and the design of the motion plan on a low-dimensional abstraction manifold. The dimension of the abstract state is independent on the number (and identities) of robots. The second key idea is the synthesis of individual robot controllers (behaviors) that rely on feedback of individual state and the abstract state to produce the desired emergent group behavior. We illustrate these ideas with a team of a small number of indoor mobile robots. Our future work is directed toward the generation of more complex emergent behaviors and controllers for air-ground coordination in outdoor environments.

Acknowledgments

We gratefully acknowledge the support of DARPA MARS grant NBCH1020012, AFOSR grant F49620-01-1-0382, and CNPq-Brazil grant 200765/01-9.

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