Administrivia

• HW 2 due today
• Should be reading Ch. 4
Lecture 6: Reminder

1. Complete supernodes
2. Review circuits in context of lab: measuring voltage and current
3. Linear and nonlinear circuits: LED
4. Physics of the resistor (revisit Ohm’s law)
5. Equivalent resistance
Lecture 7 (today): What you should know at end of this lecture

1. Mesh current analysis (2 examples)
2. Linearity, superposition
3. Dependent sources, transistor model circuits
4. Thevenin and Norton theorems
5. Equivalent resistance (max power transfer)
Mesh current analysis
Mesh current analysis

\[
\overline{A} \cdot \overline{X} = \overline{b}
\]

\[
\overline{X} = \overline{A}^{-1} \cdot \overline{b}
\]
Mesh current analysis

**Step 1** Identify a mesh current with every mesh and a voltage across every circuit element.

**Step 2** Write KVL connection constraints in terms of the element voltages around every mesh.

**Step 3** Use KCL and the $i-v$ relationships of the elements to express the element voltages in terms of the mesh currents.

**Step 4** Substitute the element constraints from step 3 into the connection constraints from step 2 and arrange the resulting equations in standard form.
Planar meshes
Example
(Ex 3.5 in book, 6th ed)

Find the branch currents $I_1$, $I_2$, and $I_3$ using mesh analysis.

1. $-15V + i_1 \cdot 5\Omega + (i_3 \cdot 10\Omega + 10V) = 0$

2. $-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$

3. $15i_1 - 10i_2 - 5 = 0$

4. $-10 + (i_2 - i_1)10 + 6i_2 + 4i_2 = 0$

5. $-10i_1 + 20i_2 - 10 = 0$

\[
\begin{bmatrix}
15 & -10 \\
-10 & 20
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}
= 
\begin{bmatrix}
5 \\
10
\end{bmatrix}
\]
\[ V = v_g \]

\[ R_4 = 10k \]

\[ 1V = v_x \]

\[ R_2 = 10k \]

\[ \beta v_x = 100A \cdot 1V = 100A \]

\[ R_3 = 100\Omega \]

\[ v_o = 100A \cdot 100\Omega = 10kV \]

\[ L_0 v_{CCS} \]

\[ \beta = 100/\Delta_0 \]
Example

Use mesh analysis to find the current $I_o$ in the circuit

1. $-24 + 10(i_1 - i_2)$
2. $24(i_1) + 4(i_2) = 0$
3. $12(i_3 - i_1) + 4(i_3 - i_2) - 4(i_1 - i_2) = 0$

$I_o = i_1 - i_2$
Example

Mesh analysis with a current source...

\[ i_2 = -5A \]

1. \[ -10 + 4i_1 + 6(i_1 - i_2) = 0 \]
2. \[ 6(i_2 - i_1) + 3i_2 + ? = 0 \]

KCL: \( \left( \frac{1}{R_2} + \frac{1}{R_3} \right) v_A - \frac{1}{R_2} v_C - \frac{1}{R_3} v_D + \frac{1}{R_1} (v_A + 10) - \frac{1}{R_1} v_E = 0 \)
Example

Mesh analysis with a current source between two mesh panels.

\[ i_1 + 6A = i_2 \quad \rightarrow \quad i_1 = i_2 - 6A \]

\[ \text{KVL: } -20 + 6i_1 + 10i_2 + 4i_2 = 0 \]
\[ (i_1+6) \quad (i_1+6) \]
\[ (6+10+4)i_1 - 20 + 60 + 24 = 0 \]
\[ 20i_1 + 44 = 0 \]
\[ i_1 = \frac{-44}{20} = -2.2 \text{A} \]
Dependent sources and DC Transistor Circuits
Transistors

- Bipolar junction transistor (BJT)
- Field effect transistor (FET)
Examples

Bipolar junction transistor

MOS Field Effect transistor

\[ v_x = I_0 R_i \]

\[ v_0 = G v_x = G I_0 R_i \]

\[ v_L = \frac{R_L}{R_0 + R_L} v_0 \]
Transistor Model

**Modes:**
- Active $\Rightarrow$
- Cutoff
- Saturation

**Active mode:** $v_{BE} \sim 0.7V$, $I_C = \beta I_B$, $\beta$ is current gain

**KL:**
$I_E = I_B + I_C$

$I_C = \beta I_B$
Transistor Model

\[
\begin{align*}
I_B &\rightarrow \quad \text{CCCS (Current Controlled Current Source)} \\
V_{BE} &\rightarrow \quad \beta I_B \\
V_{CE} &\rightarrow \quad + \\
C &\rightarrow \quad -
\end{align*}
\]
Transistor Example

Find $I_B$, $I_C$ and $v_o$. Assume active mode and $\beta = 50$.

\[ i_B = \frac{4V - 0.7V}{200k} = \frac{3.3V}{200k} = \frac{16.7}{1000} = 16.7 \mu A \]

\[ i_c = \beta i_B = \frac{1670}{2} \mu A = 835 \mu A \]

\[ v_o = 6V - 100i_c = 6V - 100 \times 835 \mu A = 6V - 83.5mV = 6V - 0.0835V = 5.9165V \]
Transistor Example

Find $I_\text{B}$, $I_\text{C}$ and $v_\text{o}$. Assume active mode and $\beta = 50$. 
Linearity, superposition, source transformation
Linearity

• Any output is proportional to any input
Linearity: example

Plot $v_o$ vs. $i_s$. Try $i_s = 30\, A, 45\, A$
Superposition

• For multiple inputs (e.g. sources):
  • Any output is the sum of outputs due to each input turned on separately
  • “Turned off input” means \( V=0 \) for voltage sources, \( I=0 \) for current sources.
  • Dependent sources stay.

\[
\begin{align*}
\text{\textbullet} & \quad v_s = 0 \quad \text{(off)} \\
\text{\textbullet} & \quad i_s = 0 \quad \text{(off)}
\end{align*}
\]
Superposition: example 1

- Use superposition to find $v$. 
Use superposition to find $v_x$ in the circuit of **Fig. 4.11**.

![Circuit Diagram](image)

**Figure 4.11** For **Practice Prob. 4.4**.