Stochastic Radiative Transfer in Species

Mixtures & Time Dependent SRT

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Chapter 6

Stochastic Radiative Transfer in Species Mixtures & Time Dependent SRT

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1. Introduction

Chapter 7 introduced the basic principles of the stochastic RT approach, which describes stationary radiation regime in single species discontinuous vegetation canopies. While this is 1D approach, it captures 3D radiation effects. The model parameterization is convenient for operational remote sensing applications. However, the stochastic model allows further enhancements to suite the needs of specialized applications to characterization of 3D canopy structure. Two such applications are detailed in this Chapter: remote sensing of mixtures of vegetation species and lidar remote sensing of vertical vegetation structure.

2. Stochastic Radiative Transfer in a Mixture of Species

2.1. Remote Sensing of Mixtures

The problem of remote sensing of mixture of vegetation species is known as a scaling issue, that is, given biophysical parameters and radiation field over pure species at sub-pixel scale one needs to estimate those parameters at the scale of a mixed pixel (Fig.1). Multiple approaches were developed to address the scaling issue, which can be grouped into two basic categories:

empirical/statistical and physically-based. The approaches from the first category are widely used for sub-pixel land cover characterization: linear mixture models [DeFries et al., 1999], neural networks [Carpenter et al., 1999], Gaussian mixture discriminant analysis [Ju et al., 2003], decision trees [McIver & Friedl, 2002] and others. The key idea of the above methods is to model satellite measured radiation over a mixed pixel as a weighted sum of the radiation fields over pure land cover classes. Linear and non-linear models were implemented to retrieve the unknown weights, corresponding to the proportion of pure land cover classes in the mixed pixel. The linear mixture approach was implemented operationally to generate global vegetation continuous fields product from NASA's MODerate resolution Imaging Spectroradiometer (MODIS) measurements [WWW1; DeFries et al., 1999]. It was noted, however, that species in a mixture may exhibit significant degree of radiative interaction, which may bias retrievals especially in the case of linear models [Borel & Gerstl, 1994].



Figure 1. Natural species mixture at NOBS (Northern Old Black Spruce) boreal forest site in northern Manitoba, Canada (55° 52' 46.632" N, 98° 28' 50.916" W). Panel a): 4-m resolution IKONOS false-color image over NOBS site for July 23, 2000. Panel b) 30-m resolution Land cover map over the same area for year 2000. (Images credit: BigFoot team).

In contrast to the empirical methods, physically-based approaches describe in details the physical processes of interaction of radiation with canopy at the level of elementary volume of vegetation. Optical properties of a mixture in such volume are represented as weighted average of optical properties of pure species. The RT equation is used to model the radiation field with effective optical properties of mixed canopy. The above modeling principles were implemented, for instance, in the scaling scheme of the radiation block of the Common Land Model (CLM) [WWW2, Tian et al., 2004] and the MODIS Leaf Area Index (LAI) algorithm [WWW3, Tian et al., 2002]. The major limitation of the above schemes is that they are based on the turbid medium mixture approximation, where canopy is represented as a mixed gas of vegetation species and

gaps. With the turbid medium approach, one major feature of the natural vegetation is missingspatial structure of a mixture, which may substantially affect radiation regime. Additional closely related theoretical work based on RT equation is the linear kinetic theory of stochastic mixture in cloudy atmospheres, developed by Pomraning [1991].

Below we detail application of the stochastic approach to model radiation regime in a mixture of discontinuous vegetation canopies. The stationary Stochastic RT (SRT) equation for single species (Chapter 7) was extended into the Stochastic Mixture RT (SMRT) equation for multiple species. In general, natural heterogeneity of vegetation (3D canopy structure) has two major components: discontinuity of vegetation; and mixture of species. The stochastic model for single species (Chapter 7) deals only with the first component of heterogeneity, discontinuity of vegetation, while the stochastic mixture model provides complete solution to the problem. The possibility of extending the original stochastic model to the mixture model already can be seen in the fact that single species discontinuous canopy can be treated as a mixture- a mixture of vegetation and gaps. The presentation is organized as follows. We start with the 3D RT equation and required parameterization of structure and optics of mixed canopy structure. Next, we perform an analytical and numerical analysis of the 3D radiation effects simulated by the SMRT model.

2.2. Model Formulation

3D RT Equation: Consider a heterogeneous vegetation canopy (a mixture of N different vegetation species and gaps as shown in Fig. 2), eliminated form above by solar radiation (both direct and diffuse components). The radiation regime in this system is described by the stationary 3D transport equation for radiation intensity, $I(\underline{r}, \underline{\Omega})$, at spatial location \underline{r} and direction $\underline{\Omega}$:

$$\underline{\Omega}\nabla I(\underline{r},\underline{\Omega}) + \sigma(\underline{r},\underline{\Omega}) I(\underline{r},\underline{\Omega}) = \int_{4\pi} d\underline{\Omega}' \,\sigma_{S}(\underline{r},\underline{\Omega}' \to \underline{\Omega}) I(\underline{r},\underline{\Omega}'), \tag{1}$$

where $\sigma(\underline{r},\underline{\Omega})$ is the extinction coefficient, and $\sigma_s(\underline{r},\underline{\Omega}' \to \underline{\Omega})$ is the differential scattering coefficient, detailed later in this Section. The unique solution of the Eq. (1) is specified by boundary conditions,

$$\begin{cases} I(z = 0, \underline{\Omega}) = \frac{f_{dir}(\underline{\Omega}_{0})}{\left|\mu(\underline{\Omega}_{0})\right|} \delta(\underline{\Omega} - \underline{\Omega}_{0}) + [1 - f_{dir}(\underline{\Omega}_{0})] d(\underline{\Omega}, \underline{\Omega}_{0}), \mu < 0, \\ I(z = H, \underline{\Omega}) = \frac{\rho_{soil}}{\pi} \int_{2\pi^{-}} d\underline{\Omega}' I(z = H, \underline{\Omega}') \left|\mu(\underline{\Omega}')\right|, \mu > 0, \end{cases}$$
(2)

where the first equation specifies incoming direct, $\delta(\underline{\Omega} - \underline{\Omega}_0)$, and diffuse, $d(\underline{\Omega}, \underline{\Omega}_0)$, radiation at the top of canopy ($\underline{\Omega}_0$ is the direction of solar beam), and $f_{dir}(\underline{\Omega}_0)$ denotes the ratio of direct to total incoming solar flux. The second equation specifies boundary condition at the canopy bottom, soil surface, which is assumed to be a Lambertian surface with hemispherical reflectance, ρ_{soil} .

Figure 2. Schematic plot of mixture of discontinuous vegetation canopies (broadleaf and needle leaf species) in a coordinate system. The vertical axis, Z, is directed down. Canopy height is H. The angular direction, θ , is measured relative to the upward direction.

The spatial structure of heterogeneous mixed canopy can be characterized by the indicator function of a canopy, $\chi(\underline{r})$, defined for each spatial location, \underline{r} , as follows:

$$\chi(\underline{\mathbf{r}}) = \sum_{j} \chi^{(j)}(\underline{\mathbf{r}}), \tag{3a}$$

where $\chi^{(j)}(\underline{\mathbf{r}})$ is an indicator function of the individual species 'j':

$$\chi^{(j)}(\underline{\mathbf{r}}) = \begin{cases} 1, & \text{if } \underline{\mathbf{r}} \in \text{species "} j^{"}, \ j = 1, N, \\ 0, & \text{otherwise.} \end{cases}$$
(3b)

The equations above specify overall architecture of vegetation canopy as cumulative contribution of individual species in a mixture. The indicator function is assumed to be a random variable. We further assume that a particular spatial location is occupied only by a single species, i.e.,

$$\chi^{(i)}(\underline{\mathbf{r}}) \cdot \chi^{(j)}(\underline{\mathbf{r}}) = 0, \, \mathbf{i} \neq \mathbf{j}.$$
(3c)

Density of canopy is defined by the Leaf Area Index (LAI) – one-sided green leaf area per unit ground area $[m^2/m^2]$. In the case of mixture of species,

$$LAI = \frac{1}{S} \int_{V} d\underline{r} \ d_{L}(\underline{r}) = \sum_{j} \frac{1}{S} \int_{V} d\underline{r} \ d_{L}^{(j)} \chi^{(j)}(\underline{r}) = \sum_{j} LAI^{(j)},$$
(4)

where $d_L^{(j)}$ and LAI^(j) are one-sided foliage area volume density $[m^2/m^3]$ and LAI of species 'j', respectively, and the integration is performed over a volume of canopy, V, with a footprint, S. Given the indicator function, we can define spatially varying extinction coefficient, $\sigma(\underline{r},\underline{\Omega})$, and differential scattering coefficient, $\sigma_s(\underline{r},\underline{\Omega}' \rightarrow \underline{\Omega})$, as follows

$$\sigma(\underline{\mathbf{r}},\underline{\Omega}) = \sum_{j} \sigma^{(j)}(\underline{\Omega}) \chi^{(j)}(\underline{\mathbf{r}}) = \sum_{j} d_{L}^{(j)} \chi^{(j)}(\underline{\mathbf{r}}) \ \mathbf{G}^{(j)}(\underline{\Omega}), \tag{5}$$

$$\sigma_{\rm S}(\underline{r},\underline{\Omega'}\to\underline{\Omega}) = \sum_{\rm j} \sigma_{\rm S}^{(\rm j)}(\underline{\Omega'}\to\underline{\Omega}) \chi^{(\rm j)}(\underline{r}) = \sum_{\rm j} \frac{d_{\rm L}^{(\rm j)}\chi^{(\rm j)}(\underline{r})}{\pi} \Gamma^{(\rm j)}(\underline{\Omega'}\to\underline{\Omega}), \tag{6}$$

where $G^{(j)}(\underline{\Omega})$ is the mean projection of leaf normals in the direction $\underline{\Omega}$ and $\Gamma^{(j)}(\underline{\Omega}' \to \underline{\Omega})$ is the area scattering phase function for species 'j' [Ross, 1981]. The above parameters depend on the probability density of species leaf normal orientation, $g_L^{(j)}(\underline{r}, \underline{\Omega}_L)$, ($\underline{\Omega}_L$ is a leaf normal direction) and species spectral leaf albedo, $\omega^{(j)}(\lambda)$ (λ is a wavelength) [Ross, 1981]. Finally, note that in the case of singe species, canopy indicator function is equivalent to a single species indicator function, and thus mixture model reduces to a model for single heterogeneous species, detailed in Chapter 7.

Stochastic Mixture RT Equations: The mathematical formulation of the stochastic mixture RT equation requires two types of averages: (1) $U^{(i)}(z, \underline{\Omega})$, mean intensity over the portion of the horizontal plane at depth *z*, occupied by species 'i'; (2) $\overline{I}(z, \underline{\Omega})$, mean intensity over the total space of the horizontal plane at depth *z*,

$$U^{(i)}(z,\underline{\Omega}) = \lim_{R \to \infty} \frac{1}{S_R \cap T_z^{(i)}} \iint_{S_R \cap T_z^{(i)}} dx dy \ \chi^{(i)}(x,y,z) I(x,y,z,\underline{\Omega}),$$
(7)

$$\bar{I}(z,\underline{\Omega}) = \lim_{R \to \infty} \frac{1}{S_R} \iint_{S_R} dx dy \ I(x, y, z, \underline{\Omega}).$$
(8)

In the above, S_R denotes the area of a circle of radius R; $T_z^{(i)}$ denotes the area of the horizontal plane at depth z, occupied by species 'i'.

Two stochastic moments characterize structure of the mixed canopy. The first stochastic moment is the probability, p, of finding species "i" at canopy depth z,

$$p^{(i)}(z) = \lim_{R \to \infty} \frac{1}{S_R} \iint_{S_R \cap T_z^{(i)}} dx dy \chi^{(i)}(z, x, y) \equiv \lim_{R \to \infty} \frac{S_R \cap T_z^{(i)}}{S_R}.$$
(9)

The second moment is the pair-correlation function, q, between species 'i' at canopy depth z and species 'j' at depth ξ along the direction $\underline{\Omega}$,

$$q^{(i,j)}(z,\xi,\underline{\Omega}) = \lim_{R \to \infty} \frac{S_R \cap T_z^{(i)} \cap T_\xi^{(j)} \left[\frac{\Omega_x}{\Omega_z} (z-\xi), \frac{\Omega_y}{\Omega_z} (z-\xi) \right]}{S_R}.$$
 (10)

In the above, Ω_x , Ω_y , and Ω_z are projections of a unit direction vector, $\underline{\Omega}$, on the x, y, and z axes, respectively. Argument for $T_{\xi}^{(j)}$ denotes a shift of the origin of plane ξ relative to plane z along x and y directions, required to evaluate correlation between the planes in direction $\underline{\Omega}$ (cf. Fig. 2). Using the first and second moments of a vegetation structure, the conditional pair-correlation of species, $K^{(i,j)}$, can be evaluated as

$$K^{(i,j)}(z,\xi,\underline{\Omega}) = \frac{q^{(i,j)}(z,\xi,\underline{\Omega})}{p^{(i)}(z)}.$$
(11)

The procedure to derive the stochastic RT equations for multiple species from 3D RT equation is similar to one for single species (Chapter 7). The main difference is that integration over space occupied by vegetation should be split into sum of integration over individual species. We skip derivations here and provide only final equations (cf. Shabanov et al. [2007] for details). The system of equation for mean intensity over total space, $\overline{I}(z, \Omega)$, is

$$\begin{cases} \bar{I}(z,\underline{\Omega}) + \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{0}^{z} d\xi \, p^{(j)}(\xi) \, \sigma^{(j)}(\underline{\Omega}) \, U^{(j)}(\xi,\underline{\Omega}) = \\ = \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{0}^{z} d\xi \int_{4\pi} d\underline{\Omega}' \, p^{(j)}(\xi) \, \sigma^{(j)}_{S}(\underline{\Omega}' \to \underline{\Omega}) \, U^{(j)}(\xi,\underline{\Omega}') + \bar{I}(0,\underline{\Omega}), \quad \mu < 0, \\ \bar{I}(z,\underline{\Omega}) + \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{z}^{H} d\xi \, p^{(j)}(\xi) \, \sigma^{(j)}(\underline{\Omega}) \, U^{(j)}(\xi,\underline{\Omega}) = \\ = \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{z}^{H} d\xi \int_{4\pi} d\underline{\Omega}' \, p^{(j)}(\xi) \, \sigma^{(j)}_{S}(\underline{\Omega}' \to \underline{\Omega}) U^{(j)}(\xi,\underline{\Omega}') + \bar{I}(H,\underline{\Omega}), \quad \mu > 0. \end{cases}$$
(12)

The system of equation for mean intensity over individual species $U^{(i)}(z, \underline{\Omega})$, i=[1,N], is

$$\begin{aligned} & \left| U^{(i)}(z,\underline{\Omega}) + \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{0}^{z} d\xi \ K^{(i,j)}(z,\xi,\underline{\Omega}) \, \sigma^{(j)}(\underline{\Omega}) \ U^{(j)}(\xi,\underline{\Omega}) = \\ & = \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{0}^{z} d\xi \int_{4\pi} d\underline{\Omega}' \ K^{(i,j)}(z,\xi,\underline{\Omega}) \, \sigma^{(j)}_{S}(\underline{\Omega}' \to \underline{\Omega}) U^{(j)}(\xi,\underline{\Omega}') + U^{(i)}(0,\underline{\Omega}), \quad \mu < 0, \\ & \left| U^{(i)}(z,\underline{\Omega}) + \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{z}^{H} d\xi \ K^{(i,j)}(z,\xi,\underline{\Omega}) \, \sigma^{(j)}(\underline{\Omega}) U^{(j)}(\xi,\overline{\Omega}) = \\ & = \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{z}^{H} d\xi \int_{4\pi} d\underline{\Omega}' \ K^{(i,j)}(z,\xi,\underline{\Omega}) \, \sigma^{(j)}_{S}(\underline{\Omega}' \to \underline{\Omega}) U^{(j)}(\xi,\underline{\Omega}') + U^{(i)}(H,\underline{\Omega}), \quad \mu > 0. \end{aligned}$$
(13)

In the Eq. (12) (Eq. (13)), $\overline{I}(0,\Omega)$ and $\overline{I}(H,\Omega)(U^{(i)}(0,\Omega))$ and $U^{(i)}(H,\Omega))$ denote mean radiation intensities over whole horizontal plane (individual species) at the canopy boundaries; in the typical case of the uniform boundary conditions they are equal to the corresponding 3D values, $I(z = 0,\Omega)$ and $I(z = H,\Omega)$ (cf. Eq. (2)). According to Eq. (12), $\overline{I}(z,\Omega)$, depends on $U^{(i)}(z,\Omega)$, i=[1,N]. Note that, in contrast to a single equation for single species (Chapter 7), the Eq. (13) for N species corresponds to systems of N equations. This accounts for the fact of species radiative coupling.

Separation of Direct and Diffuse Radiation: The mean intensity over species, $U^{(i)}(z, \vec{\Omega})$, can be decomposed into the direct, $U^{(i)}_{\delta}(z)$, and diffuse, $U^{(i)}_{d}(z, \underline{\Omega})$, components, according to the pattern of incoming solar radiation, Eq. (2), namely,

$$U^{(i)}(z,\underline{\Omega}) = \frac{f_{dir}(\underline{\Omega}_0)}{\left|\mu(\underline{\Omega}_0)\right|} U^{(i)}_{\delta}(z)\delta(\underline{\Omega}-\underline{\Omega}_0) + U^{(i)}_{d}(z,\underline{\Omega}).$$

Substituting this decomposition into Eq. (13) and collecting terms, which contain the Dirac's delta function, $\delta(\underline{\Omega} - \underline{\Omega}_0)$, we will get system of N equations for the direct component, $U_{\delta}^{(i)}(z)$:

$$U_{\delta}^{(i)}(z) + \frac{1}{\left|\mu(\underline{\Omega}_{0})\right|} \sum_{j=0}^{z} d\xi \ K^{(i,j)}(z,\xi,\underline{\Omega}_{0}) \ \sigma^{(j)}(\underline{\Omega}_{0}) \ U_{\delta}^{(j)}(\xi) = 1,$$
(14)

and the system of N equations for the diffuse component, $U_d^{(i)}(z, \underline{\Omega})$:

$$\begin{cases} U_{d}^{(i)}(z,\underline{\Omega}) + \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{0}^{z} d\xi \ K^{(i,j)}(z,\xi,\underline{\Omega}) \ \sigma^{(j)}(\underline{\Omega}) \ U_{d}^{(j)}(\xi,\underline{\Omega}) = \\ = \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{0}^{z} d\xi \ K^{(i,j)}(z,\xi,\underline{\Omega}) \ S^{(j)}(\xi,\underline{\Omega}) + U_{0}^{(i)}(z,\underline{\Omega},\underline{\Omega}_{0}), \quad \mu < 0, \\ U_{d}^{(i)}(z,\underline{\Omega}) + \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{z}^{H} d\xi \ K^{(i,j)}(z,\xi,\underline{\Omega}) \ \sigma^{(j)}(\vec{\Omega}) \ U_{d}^{(j)}(\xi,\underline{\Omega}) = \\ = \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{z}^{H} d\xi \ K^{(i,j)}(z,\xi,\underline{\Omega}) \ S^{(j)}(\xi,\underline{\Omega}) + U_{H}^{(i)}(z,\underline{\Omega},\underline{\Omega}_{0}), \quad \mu > 0, \end{cases}$$
(15a)

where,

$$S^{(j)}(\xi,\underline{\Omega}) = \int_{4\pi} d\underline{\Omega}' \ \sigma_{S}^{(j)}(\underline{\Omega}' \to \underline{\Omega}) U_{d}^{(j)}(\xi,\underline{\Omega}'), \qquad (15b)$$

$$U_{0}^{(i)}(z,\underline{\Omega},\underline{\Omega}_{0}) = \frac{f_{dir}(\underline{\Omega}_{0})}{|\mu(\underline{\Omega})\mu(\underline{\Omega}_{0})|} \sum_{j} \int_{0}^{z} d\xi \ K^{(i,j)}(z,\xi,\underline{\Omega}) \sigma_{S}^{(j)}(\underline{\Omega}_{0} \to \underline{\Omega}) U_{\delta}^{(j)}(\xi) + [1 - f_{dir}(\underline{\Omega}_{0})] d(\underline{\Omega},\underline{\Omega}_{0}), \qquad (15c)$$

$$U_{\rm H}^{(i)}(z,\underline{\Omega},\underline{\Omega}_0) = \frac{f_{\rm dir}(\underline{\Omega}_0)}{\left|\mu(\underline{\Omega})\mu(\underline{\Omega}_0)\right|} \sum_{j} \int_{z}^{\rm H} d\xi \ K^{(i,j)}(z,\xi,\underline{\Omega}) \ \sigma_{\rm S}^{(j)}(\underline{\Omega}_0 \to \underline{\Omega}) U_{\delta}^{(j)}(\xi) + U^{(i)}({\rm H},\underline{\Omega}). \tag{15d}$$

The mean intensity over the total space of the horizontal plane, $\bar{I}(z, \Omega)$, can be decomposed into direct, $\bar{I}_{\delta}(z)$, and diffuse, $\bar{I}_{d}(z, \Omega)$, components similarly:

$$\bar{I}_{\delta}(z) = 1 - \frac{1}{|\mu(\underline{\Omega}_{0})|} \sum_{j=0}^{z} \int_{0}^{z} d\xi \ p^{(j)}(\xi) \sigma^{(j)}(\underline{\Omega}_{0}) U_{\delta}^{(j)}(\xi),$$
(16)

and

$$\begin{cases} \bar{I}_{d}(z,\underline{\Omega}) = -\frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{0}^{z} d\xi \ p^{(j)}(\xi) \sigma^{(j)}(\underline{\Omega}) U_{d}^{(j)}(\xi,\underline{\Omega}) + \\ + \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{0}^{z} d\xi \ p^{(j)}(\xi) S^{(j)}(\xi,\underline{\Omega}) + U_{0}^{(i)}(z,\underline{\Omega},\underline{\Omega}_{0}), \quad \mu < 0, \\ \bar{I}_{d}(z,\underline{\Omega}) = -\frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{z}^{H} d\xi \ p^{(j)}(\xi) \sigma^{(j)}(\underline{\Omega}) U_{d}^{(j)}(\xi,\underline{\Omega}) + \\ + \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \int_{z}^{H} d\xi \ p^{(j)}(\xi) S^{(j)}(\xi,\underline{\Omega}) + U_{H}^{(i)}(z,\underline{\Omega},\underline{\Omega}_{0}), \quad \mu > 0, \end{cases}$$
(17)

where, $S^{(j)}(\xi, \underline{\Omega})$, $U_0^{(i)}(z, \underline{\Omega}, \underline{\Omega}_0)$ and $U_H^{(i)}(z, \underline{\Omega}, \underline{\Omega}_0)$ are defined by Eqs. (15b-d).

Absorptance: The terms of the energy conservation law (canopy albedo, absorptance and transmittance) and mean intensities ($\overline{I}(z, \Omega)$ and $U^{(i)}(z, \Omega)$), derived according to the stochastic approach, follow the general rules for the BS- and S- problems (Chapter 7). The equation for absorptance has special features as it explicitly accounts for the contribution of different species, characterized by different optical properties. Taking into account the definition of absorptance (cf. Chapter 7), and Eqs. (7) and (9), the absorptance of the multi-species vegetation canopy is

$$A = \frac{1}{S_{R}} \iint_{Vx4\pi} d\underline{r} d\underline{\Omega} (1 - \omega(\underline{r})) \sigma(\underline{r}, \underline{\Omega}) I(\underline{r}, \underline{\Omega})$$
$$= \frac{1}{S_{R}} \int_{0}^{H} dz \int_{4\pi} d\underline{\Omega} \int_{\pi^{2}} dx dy \sum_{j} \chi^{(j)}(\underline{r}) (1 - \omega^{(j)}) \sigma^{(j)}(\underline{\Omega}) I(\underline{r}, \underline{\Omega})$$

$$=\sum_{j}(1-\omega^{(j)})\int_{0}^{H}dz\int_{4\pi}d\underline{\Omega} \frac{S_{R}\cap T_{z}^{(j)}}{S_{R}}\cdot\frac{1}{S_{R}\cap T_{z}^{(j)}}\int_{S_{R}}dxdy\chi^{(j)}(\underline{r})\sigma^{(j)}(\underline{\Omega})I(\underline{r},\underline{\Omega})$$
$$=\sum_{j}(1-\omega^{(j)})\int_{0}^{H}dz\int_{4\pi}d\underline{\Omega} p^{(j)}(z)\sigma^{(j)}(\underline{\Omega})U^{(j)}(z,\underline{\Omega}).$$
(18)

Taking into account Eq. (18), we evaluate absorptance of the mixture as a linear sum of absorptances of individual species for BS- and S- problems,

$$A \approx \sum_{j} A_{BS}^{(j)} + \frac{\rho_{soil}}{1 - \rho_{soil} R_{S}} T_{BS} A_{S}^{(j)} .$$
 (19)

Conditional Pair-correlation Function of Mixture of Species: The critical parameters of the SMRT equations are two stochastic moments of a canopy structure: the probability of finding species, $p^{(i)}$ (Eq. (9)), and the conditional pair-correlation of species, $K^{(i,j)}$ (Eq. (11)). The following three classes of canopy structure can be identified: a) non-ordered/chaotic mixture of species and gaps or turbid medium; b) ordered mixture of species without gaps; c) ordered mixture of species with gaps. The canopy structure (order/chaoticity) is controlled by the conditional pair-correlation of species, $K^{(i,j)}$, while amount of gaps is controlled by the probability of finding species $p^{(i)}$. In the general case, $K^{(i,j)}$ satisfies the following symmetry condition:

$$p^{(i)}(z) \cdot K^{(i,j)}(z,\xi,\underline{\Omega}) \equiv S_{R} \cap T_{z}^{(i)} \cdot \frac{S_{R} \cap T_{z}^{(i)} \cap T_{\xi}^{(j)}}{S_{R} \cap T_{z}^{(i)}}$$
$$= S_{R} \cap T_{\xi}^{(j)} \cdot \frac{S_{R} \cap T_{\xi}^{(j)} \cap T_{z}^{(j)}}{S_{R} \cap T_{\xi}^{(j)}}$$
$$\equiv p^{(j)}(\xi) \cdot K^{(j,i)}(\xi,z,-\underline{\Omega}).$$
(20)

The additional properties of $K^{(i,j)}$ in the special cases of canopy structure are as follows. In the case of turbid medium there is no correlation between phytoelements of different species, and therefore, $K^{(i,j)}$ simplifies:

$$K^{(i,j)}(z,\xi,\underline{\Omega}) = \frac{q^{(i,j)}(z,\xi,\underline{\Omega})}{p^{(i)}(z)} = \frac{p^{(i)}(z)p^{(j)}(\xi)}{p^{(i)}(z)} = p^{(j)}(\xi).$$
(21)

In the case of ordered species without gaps, K^(i,j) satisfies the following two constraints:

$$\sum_{j} K^{(i,j)}(z,\xi,\underline{\Omega}) \equiv \sum_{j} \frac{S_{R} \cap T_{z}^{(i)} \cap T_{\xi}^{(j)}}{S_{R} \cap T_{z}^{(i)}} = 1,$$
(22)

$$\sum_{i} p^{(i)}(z) \cdot K^{(i,j)}(z,\xi,\underline{\Omega}) \equiv \sum_{i} \frac{S_{R} \cap T_{z}^{(i)}}{S_{R}} \cdot \frac{S_{R} \cap T_{z}^{(i)} \cap T_{\xi}^{(j)}}{S_{R} \cap T_{z}^{(i)}} = p^{(j)}(\xi),$$
(23)

if and only if,

Figure 3. The conditional pair-correlation function, $K^{(i,j)}$, of the SMRT model as function of horizontal distance of correlation, Δ , normalized by three diameter, 2a. The parameters are as follows: two species with probabilities $p^{(1)}=0.4$ and $p^{(2)}=0.6$; ; tree radius a=0.15; canopy height H=1.

Finally, consider the case of ordered mixture of species with gaps. The conditional paircorrelation function was derived according to the theory of stochastic geometry [Stoyan, Kendall and Mecke, 1995]. The derivations in the case of mixture of species [Shabanov et al., 2007] extend those for single species [Huang et al., 2007]. The derivations are based on the following assumptions about 3D stochastic canopy structure: a) tree species are modeled as identical cylinders; b) distribution of the tree centers follows stationary Poisson point process [Stoyan, Kendall and Mecke, 1995]. Under the above assumptions, the conditional pair-correlation function is as follows:

$$K^{(i,j)}(\Delta) = \begin{cases} [2p^{(i)} - 1 + (1 - p^{(i)})^{2 - \Xi(\Delta)/\pi a^2}] / p^{(i)}, i = j, \\ 1 - (1 - p^{(i)})^{1 - \Xi(\Delta)/\pi a^2}, i \neq j, \end{cases}$$
(24a)

where

$$\Xi(\Delta) = 2a^{2} \left[\arccos\left(\frac{\Delta}{2a}\right) - \frac{\Delta}{2a} \sqrt{1 - \left(\frac{\Delta}{2a}\right)^{2}} \right] \Theta(2a - \Delta).$$
 (24b)

In the above equations, parameter "a" denotes a tree radius, $p^{(i)}$ is the canopy depth independent probability of finding species, $\Theta(x)$ is the Heaviside step function, and Δ is the horizontal distance of correlation,

$$\Delta = \left| (z - \xi) \tan(\underline{\Omega}) \right|. \tag{24c}$$

The conditional pair-correlation function for two species is shown in Fig. 3 In the case of the same species, correlation decreases as distance increases. This corresponds to increasing probability of one of the points being out of the same crown as distance increases. In the case of different species, correlation increases as distance increases. This corresponds to increasing probability of two points to be located in different crowns of different species with increasing distance. In the case of short distances, within-species correlation is 1, while between-species correlation is 0. In the case of large distances, correlation between any species is vanishing and $K^{(i,j)}$ converges to $p^{(i)}$. Both limiting cases are intuitively expected and captured by the proposed model.

Numerical Scheme: The scheme for the stochastic mixture RT equations is an extension of one for single species (Chapter 7, Section 5): single system of linear equations for singles species is expanded into N systems of linear equations for N species. The key steps to solve mixture equations are identical to those for single species. The mean intensity over whole horizontal plane, $\bar{I}(z,\Omega)$, is just a linear combination of integrals of mean intensities over individual species, $U^{(i)}(z,\Omega)$, i=[1,N] (Eq. (12)). The scheme to solve for $U^{(i)}(z,\Omega)$ involves two steps: 1) solve Eq. (14) for direct intensities, $U_{\delta}^{(i)}(z,\Omega_0)$; 2) solve Eq. (15) for diffuse intensities, $U_{d}^{(i)}(z,\Omega)$. Evaluation of the direct components, $U_{\delta}^{(i)}(z,\Omega_0)$, requires solution of the system of N parametric Volterra equations. Evaluation of the diffuse components, $U_{d}^{(i)}(z,\Omega)$, is based on SOSA method (Chapter 7, Section 5), at each step of iterations one needs to solve system of N parametric Volterra equations. Thus, the key difference in numerical solution of the singe species and mixture equations is that instead of single Volterra equation one needs to solve system of N such equations. The approach to solve the system of parametric Volterra equations is illustrated below with the following equation in the general form

$$Q^{(i)}(z,\underline{\Omega}) + \frac{1}{|\mu(\underline{\Omega})|} \sum_{j=0}^{z} d\xi \ K^{(i,j)}(z,\xi,\underline{\Omega}) \sigma^{(j)}(\underline{\Omega}) Q^{(j)}(\xi,\underline{\Omega}) = F^{(i)}(z,\underline{\Omega}), \quad i=[1,N].$$
(25a)

Note that Eq. (25a) represents system of N equations, $\underline{\Omega}$ is a parameter of the parametric Volterra equation, and $Q^{(i)}(z,\underline{\Omega})$ is unknown intensity of species i. The discrete form of Eq. (25a) is as follows:

$$Q^{(i)}(k,\underline{\Omega}) + \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \sum_{\ell=1}^{k} K^{(i,j)}(k,\ell,\underline{\Omega}) \,\sigma^{(j)}(\underline{\Omega}) \,Q^{(j)}(\ell,\underline{\Omega}) = F^{(i)}(k,\underline{\Omega}).$$
(25b)

For the purpose of the following derivations, let us rewrite the above equation as follows,

$$Q^{(i)}(k,\underline{\Omega}) + \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} K^{(i,j)}(k,k,\underline{\Omega}) \,\sigma^{(j)}(\underline{\Omega}) \,Q^{(j)}(k,\underline{\Omega}) = = F^{(i)}(k,\underline{\Omega}) - \frac{1}{|\mu(\underline{\Omega})|} \sum_{j} \sum_{\ell=1}^{k-1} K^{(i,j)}(k,\ell,\underline{\Omega}) \,\sigma^{(j)}(\underline{\Omega}) \,Q^{(j)}(\ell,\underline{\Omega}).$$
(26)

Equation (26) is a system of N linear equations, which can be presented in the following matrix format,

$$\widehat{\mathbf{A}}(\mathbf{k},\underline{\Omega}) \times \mathbf{Q}(\mathbf{k},\underline{\Omega}) = \mathbf{W}(\mathbf{k},\underline{\Omega}), \tag{27a}$$

where $\hat{A}(k,\Omega)$ is an N x N matrix, and $Q(k,\Omega)$ and $W(k,\Omega)$ are N-elements vectors

$$\widehat{\mathbf{A}}^{(i,j)}(\mathbf{k},\underline{\Omega}) = \begin{cases} 1 + \frac{\sigma^{(i)}(\underline{\Omega})}{|\mu(\underline{\Omega})|} \mathbf{K}^{(i,i)}(\mathbf{k},\mathbf{k},\underline{\Omega}), & \text{if } i = j, \\ \frac{\sigma^{(j)}(\underline{\Omega})}{|\mu(\underline{\Omega})|} \mathbf{K}^{(i,j)}(\mathbf{k},\mathbf{k},\underline{\Omega}), & \text{if } i \neq j, \end{cases}$$
(27b)

$$\mathbf{W}^{(i)}(\mathbf{k},\underline{\Omega}) = \mathbf{F}^{(i)}(\mathbf{k},\underline{\Omega}) - \frac{1}{\left|\mu(\underline{\Omega})\right|} \sum_{j} \sum_{\ell=1}^{k-1} \mathbf{K}^{(i,j)}(\mathbf{k},\ell,\underline{\Omega}) \,\sigma^{(j)}(\underline{\Omega}) \,\mathbf{Q}^{(j)}(\ell,\underline{\Omega}),\tag{27c}$$

$$\mathbf{Q}^{(i)}(\mathbf{k},\underline{\Omega}) = \mathbf{Q}^{(i)}(\mathbf{k},\underline{\Omega}). \tag{27d}$$

The solution of Eq. (27) is derived sequentially starting with k=1. At each step $\hat{A}(k,\Omega)$ is a known matrix, vector $W(k,\Omega)$ can be calculated from the previous step. The solution, $Q(k,\Omega) = \hat{A}^{-1}(k,\Omega) \times W(k,\Omega)$, can be derived by Gauss-Jordan, LU-decomposition or similar techniques for solving linear system of equations [Press et al., 1986]. The complete step-by-step numerical scheme is given in Shabanov et al. [2007].

2.3. Analytical Analysis

The analytical analysis of the SMRT equations is performed for the three classes of canopy structure (cf. Section 2.2): a) non-ordered mixture of species and gaps (turbid medium); b) ordered mixture of species without gaps; c) ordered mixture of species with gaps.

Turbid Medium: In the case of non-ordered mixture of vegetation species and gaps (turbid medium) $K^{(i,j)}$ is reduced to $p^{(j)}$ (cf. Eq. (21)). In this case Eq. (13) is equivalent to Eq. (12) and $\overline{I}(z,\underline{\Omega}) = U^{(i)}(z,\underline{\Omega})$ for i=1,N, which implies no variation of the radiation fluxes between

different species. Lack of between species variation of fluxes is a consequence of the lack of spatial gradient of structure in a turbid medium: the same proportion of different species occupies each spatial location at particular canopy depth. Thus, in the case of turbid medium, Eq. (12) and (13) correspond to single equation, where species-dependent optical properties are combined in *effective* optical properties of a mixture as a whole,

$$X^{(\text{eff})}(z) = \sum_{j} p^{(j)}(z) \cdot X^{(j)}(z),$$
(28)

where X corresponds to σ and σ_s , and any expressions including its components or derivatives, such as d_L or $(1-\omega) \cdot \sigma$, etc. As mentioned before (cf. Section 2.1), approach of effective optical properties was already implemented in the scaling scheme of the radiation block of the Command Land Model and the MODIS LAI algorithm. Such approach is valid only for non-ordered canopy (turbid medium) and may lead to a bias in evaluation of the radiation field of a natural structured mixture (cf. discussion later in this Section).

To conclude, consider one important implication of the general rule of Eq. (28) with respect to gaps. In the simplest case of the turbid medium of single species with gaps, the impact of gaps on RT equations is reduced to rescaling of LAI (cf. Eq. (4)-(6)):

$$d_{\rm L}^{\rm (eff)} = p^{\rm (l)} \cdot d_{\rm L}^{\rm (l)}, \quad \sigma^{\rm (eff)} = \sigma^{\rm (l)}(d_{\rm L}^{\rm (eff)})\,, \quad \sigma_{\rm S}^{\rm (eff)} = \sigma_{\rm S}^{\rm (l)}(d_{\rm L}^{\rm (eff)}).$$

However, in the more complex case of the turbid medium of multiple species with gaps (d_L , σ , and σ_s depend on species), the above simple rescaling rule is not applicable, and effective parameters should be used:

$$d_{\rm L}^{\rm (eff)} = \sum_{j} p^{(j)} \cdot d_{\rm L}^{(j)}, \quad \sigma^{\rm (eff)} = \sum_{j} p^{(j)} \cdot \sigma^{(j)}(d_{\rm L}^{(j)}), \quad \sigma_{\rm S}^{\rm (eff)} = \sum_{j} p^{(j)} \cdot \sigma_{\rm S}^{(j)}(d_{\rm L}^{(j)}),$$

Ordered Mixture without Gaps: Next, consider the special case of ordered mixture of vegetation species with no gaps between them. This case is not equivalent to a turbid medium, because spatial heterogeneity is characterized not only by ordered composition of vegetation clumps and gaps, but also by ordered composition of different species ($K^{(i,j)} \neq p^{(j)}$). In this special case some constraints for the SMRT equations can be derived. In the following derivations we assume

$$\sum_{j} p^{(j)}(z) = 1.$$
 (29)

For simplicity we perform derivations for the direct component of radiation over species, $U_{\delta}^{(i)}(z)$. Multiplying Eq. (14) by $p^{(i)}(z)$ and performing summation over index "i", we have

$$\sum_{i} p^{(i)}(z) U_{\delta}^{(i)}(z) + \frac{1}{|\mu(\underline{\Omega}_{0})|} \sum_{i} \sum_{j} \int_{0}^{z} d\xi \ p^{(i)}(z) K^{(i,j)}(z,\xi,\underline{\Omega}_{0}) \sigma^{(j)}(\underline{\Omega}_{0}) U_{\delta}^{(j)}(\xi) = \sum_{i} p^{(i)}(z)$$
(30)

Taking into account Eqs. (23) and (29), Eq. (30) can be reduced to

$$\sum_{j} p^{(j)}(z) U_{\delta}^{(j)}(z) + \frac{1}{|\mu(\underline{\Omega}_{0})|} \sum_{j} \int_{0}^{z} d\xi \ p^{(j)}(\xi) \sigma^{(j)}(\underline{\Omega}_{0}) U_{\delta}^{(j)}(\xi) = 1.$$
(31)

Note, that Eq. (31) does not uniquely specify the solution, rather it provides a general constrain for a family of solutions with different $K^{(i,j)}$. Comparing Eqs. (31) and (16), we have

$$\bar{I}_{\delta}(z) = \sum_{j} p^{(j)}(z) U_{\delta}^{(j)}(z) .$$
(32a)

Applying the above derivation technique to Eqs. (15) and (17), we will derive similar equation for total intensity (both direct and diffuse components), namely,

$$\overline{I}(z,\underline{\Omega}) = \sum_{j} p^{(j)}(z) U^{(j)}(z,\underline{\Omega}).$$
(32b)

The above equation can be easily satisfied in the special case of the turbid medium model, as radiation over individual species is the same. However this equation is applicable to a more general case of ordered mixture of species with species-dependant fluxes (cf. further discussion in this Section and Section 2.4).

Radiation over Gaps: Next, consider the general case of ordered species with gaps. In the framework of the SMRT model, gaps can be treated as a special type of vegetation species with $\sigma = \sigma_s = 0$ and $d_L = 0$. The equation for the mean intensity over gaps, $U^{(gap)}(z, \underline{\Omega})$, can be derived as follows. Consider system of N+1 species, i.e., N vegetation species and 1 "species" of gaps, such that

$$\sum_{j} p^{(j)}(z) + p^{(gap)}(z) = 1.$$
(33)

According to Eq. (32b), formulated for the above N+1 species,

$$\overline{I}(z,\underline{\Omega}) = p^{(gap)}U^{(gap)}(z,\underline{\Omega}) + \sum_{j} p^{(j)}(z)U^{(j)}(z,\underline{\Omega}).$$
(34)

Combining Eqs. (33) and (34), we will derive expression for $U^{(gap)}(z, \underline{\Omega})$, namely

$$U^{(gap)}(z,\underline{\Omega}) = \frac{1}{1 - \sum_{j} p^{(j)}(z)} \left[\overline{I}(z,\underline{\Omega}) - \sum_{j} p^{(j)}(z) U^{(j)}(z,\underline{\Omega}) \right].$$
(35)

Note, that in the case of the turbid medium, Eq. (35) is reduced to

$$U^{(gap)}(z,\underline{\Omega}) = \overline{I}(z,\underline{\Omega}) = U^{(j)}(z,\underline{\Omega}).$$
(36)

The derived equations for the mean radiation over gaps have important implications for LAI retrievals from field measurements of radiation by a range of commercial optical instruments, including LAI-2000, AccuPAR, TRAC, etc. [Breda, 2003]. Measurements are performed over gaps and retrievals are performed according to the turbid medium model, which does not differentiate between fluxes over gaps and vegetation species (Eq. (36)). In reality, radiation fluxes over individual vegetation species and gaps may show substantial variation. To achieve better accuracy, retrieval technique of the standard optical instruments needs to be reformulated in terms of the stochastic equations.

Uncollided Radiation: Next, consider the SMRT equations in the limiting case of uncollided radiation. This case can serve as an approximation for VIS wavelengths, where absorption is large and scattering is limited. In the following we will compare derivations for turbid medium and ordered medium, to further clarify impact of canopy structure on radiation regime. To simplify derivations and get analytical expression we assume that incoming radiation is purely direct (SZA=0⁰) and probabilities of species do not depend on canopy depths, i.e., $p^{(i)}(z) \equiv p^{(i)}$. First, consider the turbid medium case. Recall, that in this case $K^{(i,j)} = p^{(i)}$ (cf. Eq. (21)) and system of Eq. (14) for direct radiation, $U_{\delta}^{(i)}$, reduces to a single equation:

$$U_{\delta}(z) + \sum_{j} p^{(j)} \sigma^{(j)} \int_{0}^{z} d\xi \ U_{\delta}(\xi) = 1.$$
(37)

The solution of the above equation is

$$U_{\delta}(z) \equiv I_{\delta}(z) \equiv T(z) = \exp\left(-\sum_{j} p^{(j)} \sigma^{(j)} z\right),$$
(38)

where T(z) denotes canopy transmittance at depth z. The canopy absorptance can also be analytically evaluated (cf. Eq. (18) for $\omega^{(j)} = 0$, j=1,N) as follows,

$$A = \sum_{j} p^{(j)} \sigma^{(j)} \int_{0}^{H} d\xi \ U_{\delta}(\xi) = 1 - \exp\left(-\sum_{j} p^{(j)} \sigma^{(j)} \ H\right).$$
(39)

Next, consider the case of ordered species composition. In the following derivation we will use the conditional pair-correlation, as defined by Eqs. (24a-b). According to these equations and the specified assumptions, $K^{(i,j)} = 1$, if i=j, and $K^{(i,j)} = 0$, if i $\neq j$. In this case Eq. (14) reduces from a system of equations to a set of independent equations for individual species, namely

$$U_{\delta}^{(i)}(z) + \sigma^{(i)} \int_{0}^{z} d\xi \ U_{\delta}^{(i)}(\xi) = 1, \quad i=[1,N].$$
(40)

The solution of each equation above depends on parameters of individual species, not whole mixture,

$$U_{\delta}^{(i)}(z) = \exp(-\sigma^{(i)}z). \tag{41}$$

Equation (16) for mean intensity in this case is reduces to

$$\bar{I}_{\delta}(z) \equiv T(z) = 1 - \sum_{j} p^{(j)} \sigma^{(j)} \int_{0}^{z} d\xi \ U_{\delta}^{(j)}(\xi) = 1 - \sum_{j} p^{(j)} [1 - \exp(-\sigma^{(j)}z)].$$
(42)

The canopy absorptance is calculated as follows,

$$A = \sum_{j} p^{(j)} \sigma^{(j)} \int_{0}^{H} d\xi \ U_{\delta}^{(j)}(\xi) = \sum_{j} p^{(j)} [1 - \exp(-\sigma^{(j)}H)].$$
(43)

Figure 4. Sunlit areas at the canopy bottom is a 3D effect, arising due to radiation streaming through the gaps without interaction with canopy (Image credit: D. Ahl, University of Wosconsin).

Compare canopy transmittance for the turbid (Eq. (38)) and ordered (Eq. (42) medium. As LAI (or canopy depth, H) is increasing, the canopy transmittance converges to 0 in the case of the turbid medium, and to a gap probability in the case of the ordered medium. The ordered medium case provides a realistic description of the radiation regime, as it accounts for radiation streaming

through gaps without interaction with vegetation, such that portion of the sunlit area at the ground is equal to the gap probability (Fig. 4). Next, compare absorptance for the turbid (Eq. (39)) and ordered (Eq. (43)) medium. As LAI increases the canopy absorptance converges to 1 in the case of turbid medium and to cumulative probability of all species in the case of the ordered medium. Again, the case of ordered medium provides a more realistic description of the radiation regime, as only the portion of photons, traveling through leaves can be absorbed. Finally, comparison of the structure of equations for absorptance and transmittance indicates that in the case of turbid medium species interact significantly, while in the case of structured medium they are radiatively decoupled. Overestimation of species radiative coupling in the turbid medium model take place because this model neglects spatial clumping of species.

Linear Mixing Assumption: Multiple land algorithms utilized for estimation of land cover mixture from coarse resolution satellite data rely on the empirical model of linear mixture of species [DeFries et al., 1999]. Under this assumption canopy spectral reflectance of a mixed pixel is expressed as a linear combination of canopy spectral reflectances of pure species. The linear mixture model ignores species radiative coupling. This coupling in a natural canopy is caused by multiple scattering, that is, after interaction with phytoelements of the first species, photon is scattered into another species.

The SMRT model can be reduced to the linear mixture model, and allows analysis of empirical assumptions of the latter. Indeed, according to Eq. (34), mean radiation over mixed pixel, $\overline{I}(z, \Omega)$, is equal to a weighted average of the radiation fields over pure species, $U^{(i)}(z, \Omega)$, and gaps, $U^{(gap)}(z, \Omega)$. However, $U^{(i)}(z, \Omega)$ are coupled though system of Eq. (13). In order to derive the linear mixture model from the SMRT model one needs to break the coupling, that is, set to 0 the conditional pair-correlation function for different species, $K^{(i,j)} = 0$, when $i \neq j$. This assumption apparently violates the basic geometry constraints on $K^{(i,j)}$ (i.e., Eqs. (22)-(23) in the case of no gaps), and intuitively one may expect that non-physical decoupling may result in the violation of energy conservation law. However this is not true. To demonstrate this, consider system of Eq. (12), where $U^{(i)}(z, \Omega)$ are derived from Eq. (13), formulated for single species 'i' (more precisely, mixture of species 'i' and gaps). Equation (12) can be rewritten in a short form as

$$\begin{cases} \bar{I}^{(\text{total})}(z) - \bar{I}^{(\text{total})}(0) = \sum_{j} \Psi^{(j)}(z) , & \mu < 0, \\ \bar{I}^{(\text{total})}(z) - \bar{I}^{(\text{total})}(H) = \sum_{j}^{j} \Psi^{(j)}(z) , & \mu > 0, \end{cases}$$
(44)

where $\Psi^{(j)}(z)$ corresponds to the terms under the sign of sum over species in Eq. (12), $\overline{I}^{(\text{total})}(z)$ is the mean intensity over whole mixture at height z, and $\overline{I}^{(\text{total})}(0)$ ($\overline{I}^{(\text{total})}(H)$) corresponds to boundary value at the canopy top (bottom). The $\Psi^{(j)}(z)$ functions are available from Eq. (12), formulated for single species 'j':

$$\begin{cases} \bar{I}^{(j)}(z) - \bar{I}^{(j)}(0) = \Psi^{(j)}(z), & \mu < 0, \\ \bar{I}^{(j)}(z) - \bar{I}^{(j)}(H) = \Psi^{(j)}(z), & \mu > 0. \end{cases}$$
(45)

Combining Eqs. (44) and (45) we have

$$\begin{cases} \bar{I}^{(\text{total})}(z) = \sum_{j} [\bar{I}^{(j)}(z) - \bar{I}^{(j)}(0)] + \bar{I}^{(\text{total})}(0), & \mu < 0, \\ \bar{I}^{(\text{total})}(z) = \sum_{j} [\bar{I}^{(j)}(z) - \bar{I}^{(j)}(H)] + \bar{I}^{(\text{total})}(H), & \mu > 0. \end{cases}$$
(46)

Next, we integrate Eq. (46) over lower and upper hemispheres to evaluate reflectances, R, and transmittances, T, and substitute the results into the energy conservation law (Chapter 7),

$$R + A + (1 - \rho)T =$$

$$= \sum_{j} [R^{(j)} - R^{(j)}(H)] + R^{(\text{total})}(H) + \sum_{j} A^{(j)} + (1 - \rho) \sum_{j} [T^{(j)} - T^{(j)}(0)] + (1 - \rho)T^{(\text{total})}(0) =$$

$$= \sum_{j} [R^{(j)} + A^{(j)} + (1 - \rho)T^{(j)}] - \sum_{j} [R^{(j)}(H) + (1 - \rho)T^{(j)}(0)] + R^{(\text{total})}(H) + (1 - \rho)T^{(\text{total})}(0)$$

$$= n - \sum_{j} [R^{(j)}(H) + (1 - \rho)T^{(j)}(0)] + R^{(\text{total})}(H) + (1 - \rho)T^{(\text{total})}(0).$$
(47)

In the last transformation we utilized the fact that the energy conservation law is valid for the single-species problem, that is,

$$R^{(j)} + A^{(j)} + (1 - \rho)T^{(j)} = 1.$$

Therefore, for the special case of the BS-problem (cf. Chapter 7), we have

$$F^{(j)}(0) = F^{(\text{total})}(0) = 1, \quad F^{(j)}(H) = F^{(\text{total})}(H) = 0, \quad \rho = 0, \quad \Rightarrow \quad R + A + (1 - \rho)T = 1,$$

and for the S-problem (cf. Chapter 7),

$$F^{(j)}(0) = F^{(\text{total})}(0) = 0, \quad F^{(j)}(H) = F^{(\text{total})}(H) = 1, \quad \rho = 0, \implies R + A + (1 - \rho)T = 1$$

Finally, from the validity of the energy conservation law for BS- and S- problems it follows the validity of this law for the total problem (cf. Chapter 7). Therefore, while radiative decoupling of vegetation species is physically meaningless, it is still a mathematically valid exercise and may describe RT processes in some other medium.

To summarize, the results of this and previous sections indicate that the linear mixture model ignores, while turbid medium overestimates species radiative coupling compared to a realistic

description of the SMRT model. The effect of radiation coupling will be further studied numerically in the next section.

2.4. Numerical Analysis

In the following we numerically investigate features of the SMRT model by comparison to the turbid medium (TM) model. Both cases were implemented with the same set of stochastic equations and input parameters, except the conditional pair-correlation function: Eq. (24) was used to implement the SMRT model, and Eq. (21) for the TM model.

Figure 5. Comparison of vertical profiles of up and down radiation fluxes as simulated by the TM and SMRT models for mixture of two species and gaps. The SMRT model captures spatial variation of fluxes between species 1 (dashed line), species 2 (dotted line), and gaps (dash-dot line) and also evaluates average flux over whole mixture (solid line). The TM model estimates only average flux over whole mixture (hollow dots). The models parameters are as follows: $p^{(1)}= 0.40$, $p^{(2)}=0.20$; $d_L^{(1)}= 4$, $d_L^{(2)}= 6$; $\omega^{(1)}(\text{Red})=0.12$, $\omega^{(2)}(\text{Red})=0.20$, $\omega^{(1)}(\text{NIR})=0.90$, $\omega^{(2)}(\text{NIR})=0.60$; $\rho_{\text{soil}}(\text{Red})=\rho_{\text{soil}}(\text{NIR})=0.10$; direct incoming flux, SZA=15⁰.

To understand the overall merits of the SMRT model, consider radiation fluxes as function of canopy depth, as shown in Fig. 5. The SMRT model differentiates between radiation fluxes over

individual species, gaps, and whole mixture (corresponding to mean intensities $U^{(i)}$, $U^{(gap)}$ and \overline{I}), while the TM model provides no distinction between the above fluxes (cf. Section 2.3). According to the SMRT model, variation of the fluxes between individual vegetation species is relatively smaller compared to difference in fluxes between vegetation and gaps. Also note, that limitations of the TM model result in a bias (both overestimation and underestimation) in estimation of mean fluxes over whole mixture compared to the SMRT simulations (compare fluxes over whole mixture). Detailed analysis of the impact of various parameters on the SMRT and TM models simulations is presented with six case studies below (Figs. 6-11).

First, consider impact of Solar Zenith Angle (SZA) on canopy albedo, absorptance and transmittance as function of LAI, as simulated with the SMRT and TM models at Red and NIR wavelengths (Fig. 6). The simulations were performed with SZA of 0^0 and 60^0 . Two vegetation species with gaps were used: $p^{(1)}=0.2$, $p^{(2)}=0.3$, $p^{(gap)}=0.5$. The complete set of parameters is presented in the figure caption. In the case of SZA=0⁰, the SMRT model predicts lower albedo, substantially lower absorptance and substantially higher transmittance compared to the TM model. However, in the case of SZA= 60° both models demonstrate quite similar results. The key physical explanation for the difference between the SMRT and TM simulations at SZA=0⁰ is that the SMRT model accounts for radiation streaming through gaps without interaction with vegetation (Fig. 4). This explains results for absorptance and transmittance. Albedo is lower in the case of the SMRT compared to the TM model, because in the former case a dark soil is better exposed through gaps in a relatively bright vegetation (compare ρ_{soil} and ω). Next, note that the numerical simulations for absorptance and transmittance at Red wavelength closely follow analytical expressions, derived for uncollided radiation for SZA=0⁰ (cf. Section 2.3). For instance, in the case of high LAI, transmittance from the SMRT model approaches $1-(p^{(1)}+p^{(2)})$ (Eq. (42)), while transmittance from the TM model converges to 0 (Eq. (38)). Next, we explain results for SZA=60⁰. In this case the effect of radiation streaming is negligible: even if photon enters canopy through a gap, it will be intercepted by a lateral surface of a tree foliage. Mathematically, the reasoning is as follows: as angle is increasing, effective distance between vegetation elements, Δ , is increasing (cf. Eq. (24)), which results in convergence of the conditional pair-correlation function for ordered species to one for non-ordered species (Fig 3).

Second, consider the impact of soil reflectance on the radiation quantities (Fig. 7). Set of parameters for this case study was similar to the previous one, except SZA is fixed and equal to 0^0 , while two soil albedos (0 and 1) were used. Note the special feature of the canopy albedo for high LAI: in the case of the TM model, albedo converges to a single value, independently from soil albedo, while such convergence does not exist in the case of the SMRT model. This is another effect, associated with radiation streaming through gaps. Next, consider results for absorptance. Canopy absorptance is increasing with increasing soil albedo, and such effect is more pronounced for the SMRT model. The physical explanation for this is that some portion of the total radiation can stream through the gaps to the canopy bottom without being absorbed. If

soil is bright, this radiation is bounced back and receives a second chance to be absorbed by a canopy. However, if soil is dark, this mechanism is vanishing. Finally, consider results for transmittance. In the case of Red wavelength, soil albedo has virtually no effects on transmittance both for the TM and SMRT models. In the case of NIR wavelength, higher soil reflectance results in a higher transmittance and this effect is enhanced in the SMRT simulations due to radiation streaming through gaps.

Third, consider the impact of species composition on a radiation regime (Fig. 8). Here we used two species with optical properties roughly corresponding to broadleaf (species 1) and needle leaf (species 2) forests, which have substantial contrast both at Red and NIR wavelengths. The probability of each species $(p^{(1)} \text{ and } p^{(2)})$ was varying from 0 up to 0.6, under restriction, that total probability of all species is constant through the simulations, i.e., $p^{(1)}+p^{(2)}=0.6=$ fixed. Thus, radiation regime was evaluated for all possible combinations of two species under significant amount of gaps, $p^{(gap)} = 0.4=$ fixed. Results indicate that both the TM and SMRT models simulate continuous, fairly large variation of canopy albedo, absorptance and transmittance with respect to species composition. However, the TM model introduces a significant bias in the estimation of the above parameters due to ignoring canopy structure. Finally, note that variations of canopy albedo, absorptance and transmittance with respect to species compositions are quite close to linear at Red wavelength, but demonstrate fairly large deviation from linearity at NIR wavelength. Note that linearity for absorptance with respect to species composition can be seen in Eqs. (39) and (43) in the case of high LAI.

Fourth, consider the role of gaps in a radiation regime (Fig. 9). Simulations were performed for two species under constrain that the probabilities of both species are equal and its sum is varying between 0 (only gaps) and 1 (only vegetation). The SMRT model is not equivalent to the TM model even in the limiting case of no gaps (cf. Section 2.3). Results for canopy albedo are as follows. Albedo at Red wavelength is decreasing as gaps are replaced with vegetation, because albedo of leaves was selected to be comparable to soil albedo, however, when more leaves are introduced, they trap radiation, scattered within canopy, more efficiently. Also, canopy albedo for the TM and SMRT models are approximately equal at Red wavelength. However, at NIR wavelength albedo is increasing with decreasing amount of gaps, and the TM model predicts higher albedo compared to the SMRT model. The physical reason for this is that at NIR wavelength leaves are brighter then soil, and therefore, increasing amount of leaves will increase albedo. Additionally, vegetation gas of the TM model efficiently covers soil, compared to discontinuous canopy with gaps in the SMRT model, which results in further enhancement of albedo in the TM model compared to the SMRT model. Next, the simulations indicate that overestimation of absorptance by the TM model compared to the SMRT model is directly proportional to the gap fraction/clumping of species. Finally, results for canopy transmittance indicate that as amount of gaps is decreasing, transmittance, evaluated by the SMRT model converges to one for the TM model.

Figure 6. Impact of SZA on canopy albedo, absorptance and transmittance, as evaluated with the TM (dots) and SMRT (lines) models. Runs for direct incoming flux with SZA=0⁰ and =60⁰ are marked with solid style and style with holes, respectively. The other parameters are as follows: $p^{(1)} = 0.20$, $p^{(2)} = 0.30$; $d_L^{(1)} = d_L^{(2)} = [0.0-16.0]$; $\omega^{(1)}(\text{Red}) = 0.12$, $\omega^{(2)}(\text{Red}) = 0.20$, $\omega^{(1)}(\text{NIR}) = 0.90$, $\omega^{(2)}(\text{NIR}) = 0.60$; $\rho_{\text{soil}}(\text{Red}) = \rho_{\text{soil}}(\text{NIR}) = 0.10$.

Figure 7. Impact of soil reflectance on canopy albedo, absorptance and transmittance, as evaluated with the TM (dots) and SMRT (lines) models. Runs with soil reflectance $\rho_{soil}(\text{Red}) = \rho_{soil}(\text{NIR}) = 0.0$ and =1.0 are marked with solid style and style with holes, respectively. The other parameters are as follows: $p^{(1)}=0.20$, $p^{(2)}=0.30$; $d_L^{(1)}=d_L^{(2)}=[0.0-16.0]$; $\omega^{(1)}(\text{Red})=0.12$, $\omega^{(2)}(\text{Red})=0.20$, $\omega^{(1)}(\text{NIR})=0.90$, $\omega^{(2)}(\text{NIR})=0.60$; direct incoming flux, SZA=0⁰

Figure 8. Impact of species composition on canopy albedo, absorptance and transmittance, as evaluated with the TM (dots) and SMRT (lines) models for the case of mixture of two species. The models parameters are as follows: $p^{(1)}+p^{(2)}=0.60$; $d_L^{(1)}=4.0$, $d_L^{(2)}=6.0$; $\omega^{(1)}(\text{Red})=0.12$, $\omega^{(2)}(\text{Red})=0.20$, $\omega^{(1)}(\text{NIR})=0.90$, $\omega^{(2)}(\text{NIR})=0.60$; $\rho_{\text{soil}}(\text{Red})=\rho_{\text{soil}}(\text{NIR})=0.10$; direct incoming flux, SZA=0⁰. Shaded areas highlight absorptance of individual species in the case of the TM model simulations.

Figure 9. Impact of gaps on canopy albedo, absorptance and transmittance, as evaluated with the TM (dots) and SMRT (lines) models for the case of mixture of two species. The models parameters are as follows: $p^{(1)}=p^{(2)}=[0.0-0.5]$; $d_L^{(1)}=4.0$, $d_L^{(2)}=6.0$; $\omega^{(1)}(\text{Red})=0.12$, $\omega^{(2)}(\text{Red})=0.16$, $\omega^{(1)}(\text{NIR})=0.80$, $\omega^{(2)}(\text{NIR})=0.70$; $\rho_{\text{soil}}(\text{Red})=\rho_{\text{soil}}(\text{NIR})=0.10$; direct incoming flux (SZA=0⁰). Shaded areas highlight absorptance of individual species in the case of the TM model simulations.

In the fifth case study we probed deeper special case of mixture of species without gaps (Fig. 10). Intuitively, in the case of no gaps, ordered mixture of species is quite close to non-ordered mixture (turbid medium). In this simulation we addressed two questions: a) When ordered mixture of species is important? b) Why gaps significantly perturb radiation field of ordered mixture of species? As mentioned earlier, key feature of vegetation structure, which differentiates between ordered and non-ordered cases is a presence of spatial gradient of optical properties of a medium (such as leaf albedo, density of LAI, etc). If the medium is ordered, the spatial gradient of the optical properties should be significant enough to modify the radiation regime of the SMRT simulations with respect to the TM simulations. Optical properties of gaps constitute especially large contrast to ones of any vegetation species. This explains special role of gaps in the SMRT simulations. The variation of the optical properties of vegetation species is as follows: leaf albedo may vary by factor of 2 in majority of cases, while variation in the foliage area volume density may be arbitrary large. Our test runs (not presented here) indicate no significant difference between the SMRT and TM simulations as function of variations of leaf albedo for typical vegetation canopies. Simulations for two species with varying foliage area volume density $(d_{L}^{(1)})$ and $d_{L}^{(2)}$ indicate significant bias in the estimation of absorptance by the TM model with respect to the SMRT model when $d_L^{(2)}/d_L^{(1)}=0$, no difference when $d_L^{(2)}/d_L^{(1)}=1$, and increasing bias with respect to increasing contrast in the foliage area volume density of species (Fig. 10). Note that the TM model introduces bias not only to total absorptance but also to the partitioning of total canopy absorptance between species.

Figure 10. Impact of species LAI density on partitioning of total absorptance between individual species, as evaluated with the TM (dots) and SMRT (lines) models for the case of mixture of two species. The models parameters are as follows: $p^{(1)}=0.4$, $p^{(2)}=0.6$; $d_L^{(1)}=4.0$, $d_L^{(2)}=[0.0-16.0]$; $\omega^{(1)}(\text{Red}) = 0.12$, $\omega^{(2)}(\text{Red}) = 0.20$, $\omega^{(1)}(\text{NIR})=0.90$, $\omega^{(2)}(\text{NIR})=0.60$; $\rho_{\text{soil}}(\text{Red}) = \rho_{\text{soil}}(\text{NIR})=0.10$; direct incoming flux, SZA=0⁰. Shaded areas highlight absorptance of individual species in the case of the TM model simulations

Figure 11. Impact of species interaction on canopy absorptance, as evaluated with the TM (dots) and SMRT (lines) models. Runs for direct (SZA=0⁰), and diffuse incoming fluxes are marked with solid style and style with holes, respectively. The other parameters are as follows: $p^{(1)}=0.4$, $p^{(2)}=0.5$ or variable; $d_{L}^{(1)}=6.0$, $d_{L}^{(2)}=9.0$ or variable; $\omega^{(1)}(\text{Red})=0.12$, $\omega^{(2)}(\text{Red})=0.16$, $\omega^{(1)}(\text{NIR})=0.60$, $\omega^{(2)}(\text{NIR})=0.80$; $\rho_{soil}(\text{Red})=\rho_{soil}(\text{NIR})=0.10$.

Next, consider the sixth case study, demonstrating the effect of species radiative interaction (Fig. 11). Here we evaluated the impact of LAI changes of species 2 on the absorptance of species 1. The LAI of species 2 was constructed according to two scenarios: a) keep $p^{(2)}$ constant and vary $d_L^{(2)}$ (top two panels for Red and NIR wavelengths); b) keep $d_L^{(2)}$ constant and vary $p^{(2)}$ (lower panels). We also used two illumination conditions: purely direct incoming flux, SZA=0⁰; and purely diffuse incoming flux. The results indicate, that absorptance of species 1 decreases by factor of two both at Red and NIR wavelengths as LAI of species 2 is changing from 0 to 4 in the case of the TM model under direct illumination. In contrast, the SMRT model predicts no significant variations in the absorptance of species 1 at Red wavelength, and increase by about 15% in the case of NIR wavelength under direct illumination. In the case of diffuse illumination the SMRT and TM models predict similar interaction of species- decrease of the absorptance of the first species as LAI of the second is increasing. This last result of this case study match the results of the first case study: under diffuse illumination or low SZA, the simulations by both

models converge. Overall, this last case study demonstrates that natural mixture of discontinuous species exhibit less radiative coupling compared to the turbid medium approximation.

The final comment is on the overall performance of the SMRT model. The SMRT code is computationally compact- CPU requirements are similar to those for the solution of 1D RT code [Shabanov et al., 2000]. Computational errors increase with increasing LAI, SZA, which is typical for majority of the RT models. Additional errors, specific to the SMRT, can be accumulated under the condition of large number of species with highly varying optical properties. Presently the performance of the model was not evaluated with respect to field measurements, this should be done in the future utilizing the required set of structured parameters and radiation measurements of sufficiently high accuracy. Nevertheless, confidence in the SMRT model performance should be drawn from the fact, that the mixture model extends the one for single species, which was extensively evaluated with field measurements and was utilized in multiple applications in the past [Huang et al., 2007; Kotchenova et al., 2003; Shabanov et al., 2000, 2005].

2.5. Summary

The application of the stochastic approach for remote sensing of vegetation mixtures was introduced. The Stochastic Mixture RT (SMRT) model describes radiation regime in a heterogeneous vegetation canopy with spatially varying optical properties. The SMRT model provides a general solution of the mixture problem, which includes, as special cases, the major approximate solutions, including the linear mixture and turbid medium mixture RT models. The SMRT model solves for the radiation quantities, direct input to remote sensing/climate applications: mean fluxes over mixture and over individual species. The canopy structure is parameterized in the SMRT model in terms of two stochastic moments: the probability of finding species and the conditional pair-correlation of species. The second moment is responsible for the 3D radiation effects, namely, radiation streaming through gaps without interaction with vegetation and variation of the radiation fluxes between different species. If the within- and between- species correlation is vanishing, the SMRT model reduces to the turbid medium RT model. Namely, this situation is realized in the SMRT simulations under direct illumination with low SZA or diffuse illumination. If the between- (but not within-) species correlation is set to zero, the SMRT model reduces to the linear mixture model. The analysis of the SMRT simulations indicates that the variation of radiation fluxes between different species is proportional to the variation of optical properties of species (leaf albedo, density of foliage, etc.) Gaps introduce significant disturbance to the radiation regime in the mixed canopy as their optical properties constitute a major contrast to those of any vegetation species. Set of accurate field measurements on canopy structure and radiation is required to further assess performance of the SMRT model and to improve modeling of the pair-correlation function.

3. Time-dependent Stochastic Radiative Transfer

3.1. Lidar Remote Sensing

Active remote sensing complements and enhances passive remote sensing measurements of the 3D canopy structure. In general, signals recorded by passive sensors, operating at visible and mid-infrared wavelength, undergo influence of a range of physical factors, including atmospheric contamination, background scattering and effect of view-illumination geometry. Thus passive remote sensing measurements have to be corrected before attempting retrievals of the canopy structure. Some of the above problems can be eliminated (however new challenges arises) with the use of major types of active remote sensing instruments, including active Synthetic Aperture Radar (SAR) systems, and small and large footprint laser altimeters. Like passive optical systems, SAR polarimetry systems map well the horizontal organization of vegetation, however fail to provide direct information on its vertical distribution. In the case of the small footprint lidars, the small-diameter beams frequently miss the top of vegetation in sparse canopies, while in dense canopies, it is difficult to determine whether a particular shot has penetrated through the canopy and reached the ground. A new class of instruments, large footprint waveformrecording laser altimeters (lidars), has demonstrated a potential to significantly improve remote estimates of vertical forest structure [Drake et al., 2002a-b; Lefsky et al., 1999a-b, 2002]. The group of existing large footprints lidars includes two air-borne instruments, the formerly utilized Scanning Lidar Imager of Canopies by Echo Recovery (SLICER) and current instrument, Laser Vegetation Imaging Spectrometer (LVIS), and the space-borne Geoscience Laser Altimeter System (GLAS), onboard the ICES at satellite. Another space-borne instrument, Vegetation Canopy Lidar (VCL), is under development presently.

SLICER transmits short-duration pulses (4 ns pulse width at the half of the amplitude) with a beam divergence of approximately 2 mrad at frequency of 80 Hz. It operates in NIR wavelengths (1064 nm). The energy transmitted per pulse is about 0.7 mJ. The SLICER footprint distribution pattern on the ground is a swath consisting of five contiguous footprint lines along the direction of flight. Each footprint is approximately 9 m in diameter. The diameter of the area covered by the telescope field-of-view (FOV) is five times as much as the laser footprint diameter. The telescope FOV is aligned to seize five contiguous cross-track footprints [Harding et al., 2000; Lefsky et al., 1999a].

LVIS is an advanced version of SLICER. It emits Gaussian-shaped pulses with 10 ns pulsewidth at repetition rates of up to 500 Hz in the NIR wavelengths (1064 nm). The energy transmitted per pulse is about 5 mJ. The LVIS telescope FOV significantly exceeds that for SLICER, 7^0 versus 2^0 , which allows it to operate in a number of different modes. Footprint sizes from 1 to 80 m are possible, determined by the altitude of the airplane and the focal length of a diverging lens in the output path. The distance between adjacent footprints can be changed both along and across track. A standard sampling pattern consists of parallel 1-km wide swaths generated using 30-40 25-m wide footprints separated by 10-20 m along and across track [Blair & Hofton, 1999].

GLAS instrument onboard the ICESat platform provides continuous global measurements along the 183-day repeat tracks with a maximum cross-track separation of 15 km at the equator. Pulses are emitted at rate of 40 Hz with pulse width of 5 ns in NIR (1064 nm). Large, non-contiguous GLAS footprints with the diameter of \sim 70 m are not optimized for retrieval of vegetation structural parameters. The primary goal of the ICESat mission is to measure ice, ocean and land elevation and cloud structure. Nevertheless, as the GLAS waveforms have high vertical resolution (of 15 cm) and good signal-to-noise ratio, some vegetation structural parameters are possible to retrieve where the ground and canopy echoes can be decomposed [Zwally et al., 2002].

Figure 12. Schematic plot of lidar waveform measurements with the SLICER instrument. (Graphycs credit: Dr. Harding, NASA GSFC).

Lidar instruments operate according to the following general scheme. A short-duration laser pulse is sent from the zenith to nadir, and then the amplitude of reflected energy is recorded as a function of time. The obtained distribution of intercepted surfaces with height is referred to as *lidar waveform* (Fig. 12). The first significant return above a noise threshold is used to estimate the top of the canopy, the midpoint of the last return represents the reflection from the ground, and canopy height is calculated as the distance between these two returns [Harding et al., 2001]. The amplitude of the recorded waveform measures the strength of the return. For surfaces with similar reflective properties and geometry within a footprint, the larger amplitude indicates more canopy material.

Current applications of the large-footprint lidars fall into three main categories: remote sensing of the ground topography, measurement of the three-dimensional canopy structure, and

estimation of forest structural attributes [Lefsky et al., 2002; Dubayah et al., 2000]. Canopy height and ground elevation are the only parameters that are measured by the lidars directly. Other structural attributes biomass and *canopy height profiles* (*CHP*s), or the distribution of canopy material with height requires apriory knowledge and modeling.

The current algorithm for retrieval of CHPs from lidar-recorded signals [Lefsky et al., 1999a; Harding et al., 2001] suffers from two significant drawbacks. The algorithm was developed following the main principles of MacArthur & Horn [1969] for estimation of foliage height profiles (FHPs) in broadleaf forests. It is based on the simplifying assumptions that (1) the horizontal distribution of leaves is uniform and (2) only single scattered photons contribute to the return signal. The assumption of uniform horizontal distribution ignores foliage clumping. However, due to foliage clumping more radiation is allowed to penetrate deeper into the canopy without being intercepted by foliage [Ni-Meister et al., 2001]. Additionally, a lidar pulse at NIR wavelengths is weakly absorbed and the returned waveform is affected by multiple scattering, especially in the case of dense forests [Govaerts, 1996]. One of the effects of multiple scattering in dense canopies is related to a significant enhancement of the lower part of the signal. The main reason for this effect is that the path of multiply scattered photons is longer than the straight line path between the instrument and the target and, thus, those photons appear delayed, compared to single-scattered photons. The significance of the effect of multiple scattering in lidar measurements is currently under debate presently [Blair & Hofton, 1999; Sun & Ranson, 2000].

Below we detail application of the stochastic approach for modeling of propagation of lidar signal in heterogeneous vegetation canopies. The stationary Stochastic RT equation for single species (Chapter 7) was extended to the Time-dependent Stochastic RT (TSRT) equation. The stochastic description of the process arises naturally due to the fact that large footprint lidars measure mean pulse intensity over the extended area of their footprints. Additionally, the stochastic equations allows to account for multiple scattering, realistic representation of forest structure including clumping and gaps, and simulation of off-nadir and multi-angular observations. The presentation is organized as follows. Starting form 3D Time-dependant RT equation we formulate the time-dependant stochastic equations. Next, the sensitivity study of the simulated non-stationary radiation regime is performed. Next, simulated waveforms are evaluated with respect to SLICER measurements over coniferous and mixed-deciduous stands and the impact of multiple scattering is assessed.

3.2. Model Formulation

3D RT Equation: Consider a single species heterogeneous vegetation canopy. A lidar beam illuminates a vertical vegetation stand of the diameter of the laser footprint. Due to the small divergence angle, photons can be assumed incident parallel on the top of the stand. The radiation

regime inside the canopy is strongly time-dependent due to the short duration of a lidar pulse. The propagation of photons through vegetation is described by the time-dependent 3D transport equation for radiation intensity, $I(t, \underline{r}, \underline{\Omega})$, at time t, spatial location \underline{r} and direction $\underline{\Omega}$:

$$\frac{1}{c}\frac{\partial I(t,\underline{r},\underline{\Omega})}{\partial t} + \underline{\Omega}\nabla I(t,\underline{r},\underline{\Omega}) + \sigma(\underline{r},\underline{\Omega}) I(t,\underline{r},\underline{\Omega}) = \int_{4\pi} d\Omega' \sigma_{s}(\underline{r},\underline{\Omega'} \to \underline{\Omega}) I(t,\underline{r},\underline{\Omega'}), \quad (48)$$

where; $\sigma(\underline{r},\underline{\Omega})$ is the extinction coefficient, and $\sigma_s(\underline{r},\underline{\Omega}' \to \underline{\Omega})$ is the differential scattering coefficient [Ross, 1981]. Both coefficients are formulated in the same way as for stationary RT for singe heterogeneous species: $\sigma(\underline{r},\underline{\Omega}) = \chi(\underline{r})\sigma(\underline{\Omega})$ and $\sigma_s(\underline{r},\underline{\Omega}' \to \underline{\Omega}) = \chi(\underline{r})\sigma_s(\underline{\Omega}' \to \underline{\Omega})$ where $\sigma(\underline{\Omega})$ and $\sigma_s(\underline{\Omega}' \to \underline{\Omega})$ are coefficients for vegetated portion space, and $\chi(\underline{r})$ is a canopy indicator function (cf. Chapter 7). The unique solution of Eq. (48) is specified by *boundary and initial conditions*:

$$\begin{cases} I(t, \underline{r}_{xy}, 0, \underline{\Omega}) = \begin{cases} 0, & \underline{r}_{xy} \notin S_{f}, & \mu(\underline{\Omega}_{0}) < 0, \\ f(t) \,\delta(\underline{\Omega} - \underline{\Omega}_{0}), & \underline{r}_{xy} \in S_{f}, & \mu(\underline{\Omega}_{0}) < 0, \end{cases} \\ I(0, \underline{r}, \underline{\Omega}) = 0, & 0 < z < H, \end{cases}$$

$$I(t, \underline{r}_{xy}, H, \underline{\Omega}) = \frac{\rho_{\text{soil}}}{\pi} \int_{2\pi^{-}} d\underline{\Omega}' \, I(t, \underline{r}_{xy}, H, \underline{\Omega}') \Big| \, \mu(\underline{\Omega}') \Big|, \quad \mu(\underline{\Omega}) > 0, \end{cases}$$

$$(49)$$

where $\underline{\Omega}_0$ is the direction of incoming radiation; f(t) is a function characterizing pulse intensity; S_f is a footprint area; and $\rho_{soil}(\lambda)$ is the hemispherical reflectance of ground which is assumed to be a Lambertian surface.

Stochastic RT Equations: A procedure to derive Time-dependant Stochastic RT (TSRT) equations from time-dependant 3D equation is similar to one for stationary 3D equation for single species (Chapter 7). Following this procedure we derive equations for the mean intensities over total space, $\overline{I}(t, z, \Omega)$, and over vegetated areas, $U(t, z, \Omega)$. The system of equation for $\overline{I}(t, z, \Omega)$ is,

$$\begin{cases} \overline{I}(t, z, \underline{\Omega}) + \frac{1}{|\mu(\underline{\Omega})|} \int_{0}^{z} \sigma(\underline{\Omega}) p(\xi) U(t, \xi, \underline{\Omega}) \, d\xi + \frac{1}{|\mu(\underline{\Omega})|} \frac{1}{c} \frac{\partial}{\partial t} \int_{0}^{z} \overline{I}(t, \xi, \underline{\Omega}) d\xi \\ = \frac{1}{|\mu(\underline{\Omega})|} \int_{0}^{z} d\xi \, p(\xi) \int_{4\pi} \sigma_{s}(\underline{\Omega}' \to \underline{\Omega}) U(t, \xi, \underline{\Omega}') \, d\overline{\Omega}' + \overline{I}(t, 0, \underline{\Omega}), \quad \mu(\underline{\Omega}) < 0, \\ \overline{I}(t, z, \underline{\Omega}) + \frac{1}{\mu(\underline{\Omega})} \int_{z}^{H} \sigma(\underline{\Omega}) p(\xi) U(t, \xi, \underline{\Omega}) \, d\xi + \frac{1}{\mu(\underline{\Omega})} \frac{1}{c} \frac{\partial}{\partial t} \int_{z}^{H} \overline{I}(t, \xi, \underline{\Omega}) d\xi \\ = \frac{1}{\mu(\underline{\Omega})} \int_{z}^{H} d\xi p(\xi) \int_{4\pi} \sigma_{s}(\underline{\Omega}' \to \underline{\Omega}) U(t, \xi, \underline{\Omega}') \, d\underline{\Omega}' + \overline{I}(t, H, \underline{\Omega}), \quad \mu(\underline{\Omega}) > 0. \end{cases}$$
(50)

The system of equations for $U(t, z, \underline{\Omega})$ is,

$$\begin{cases} U(t, z, \underline{\Omega}) + \frac{1}{|\mu(\underline{\Omega})|} \int_{0}^{z} \sigma(\underline{\Omega}) K(z, \xi, \underline{\Omega}) U(t, \xi, \underline{\Omega}) d\xi + \frac{1}{|\mu(\underline{\Omega})|} \frac{1}{c} \frac{\partial}{\partial t} \int_{0}^{z} U(t, \xi, \underline{\Omega}) d\xi \\ = \frac{1}{|\mu(\underline{\Omega})|} \int_{0}^{z} d\xi K(z, \xi, \underline{\Omega}) \int_{4\pi} \sigma_{S}(\underline{\Omega}' \to \underline{\Omega}) U(t, \xi, \underline{\Omega}') d\underline{\Omega}' + \bar{I}(t, 0, \underline{\Omega}), \quad \mu(\underline{\Omega}) < 0, \\ U(t, z, \underline{\Omega}) + \frac{1}{\mu(\underline{\Omega})} \int_{z}^{H} \sigma(\underline{\Omega}) K(z, \xi, \underline{\Omega}) U(t, \xi, \underline{\Omega}) d\xi + \frac{1}{\mu(\underline{\Omega})} \frac{1}{c} \frac{\partial}{\partial t} \int_{z}^{H} U(t, \xi, \underline{\Omega}) d\xi \\ = \frac{1}{\mu(\underline{\Omega})} \int_{z}^{H} d\xi K(z, \xi, \underline{\Omega}) \int_{4\pi} \sigma_{S}(\underline{\Omega}' \to \underline{\Omega}) U(t, \xi, \underline{\Omega}') d\underline{\Omega}' + \bar{I}(t, H, \underline{\Omega}), \quad \mu(\underline{\Omega}) > 0. \end{cases}$$
(51)

The averaging procedure leads to a parameterization of the TSRT equations in terms of the probability of finding vegetation elements at height z, p(z), and the conditional pair-correlation function of vegetation elements at two layers z and in direction in direction $\underline{\Omega}$, K(z, ξ , $\underline{\Omega}$) (cf. Chapter 7). Note that the boundary source terms in Eq. (51) are the same as in Eq. (50). The equality, U(t,0, $\underline{\Omega}$) = $\overline{I}(t,0,\underline{\Omega})$, follows from the assumption of a uniform horizontal distribution of incident laser energy. The equality U(t, H, $\underline{\Omega}$) = $\overline{I}(t,H,\underline{\Omega})$ follows from the assumption of homogeneity of the ground surface.

Two important issues related to Eqs. (50)-(51) need to be discussed. First, the integration is performed over the area approximately equal to the laser footprint area with 8.4 - 9.0 m in diameter for the coniferous sites and 10.4 - 11.0 m for the deciduous sites. As this area is relatively large, radiation fluxes through the lateral boundaries can be neglected and the concept of mean intensity is appropriate. Second, the spatial distribution of laser energy across the footprint has a circular, Gaussian distribution [Harding et al., 2000]. We approximate the Gaussian distribution with a uniform distribution in order to satisfy the requirements of a stochastic approach. The total amount of energy within the footprint area remains the same. It should be noted, in general, the use of the Gaussian and uniform distributions of incident energy would lead to different forms of the return. Within the chosen approach, however, a layer within vegetation is treated as a horizontal surface with averaged parameters characterizing its reflection and transmission properties. Thus, the model is sensitive only to the total amount of radiation incident at each layer and the Gaussian distribution can be replaced with the uniform distribution.

Separation of Direct and Diffuse Radiation: The procedure to derive equations for direct and diffuse components of mean radiation intensity over total space, $I(t, z, \Omega)$, and over vegetation, $U(t, z, \Omega)$, in the case of time-dependant equations is identical to that for stationary equations (cf. Chapter 7). We express the solution of Eq. (51) for $U(t, z, \Omega)$ as the sum of direct and diffuse components, that is,

$$U(t, z, \underline{\Omega}) = U_{\delta}(t, z) \,\delta(\underline{\Omega} - \underline{\Omega}_0) + U_{d}(t, z, \underline{\Omega}) \,. \tag{52}$$

Substituting Eq. (52) into Eq. (51) and separating the terms with $\delta(\underline{\Omega} - \underline{\Omega}_0)$ results in an equation for the direct component,

$$U_{\delta}(t,z) + \frac{\sigma(\underline{\Omega}_{0})}{\left|\mu(\underline{\Omega}_{0})\right|} \int_{0}^{z} K(z,\xi,\underline{\Omega}_{0}) U_{\delta}(t,\xi) d\xi + \frac{1}{\left|\mu(\underline{\Omega}_{0})\right|} \frac{1}{c} \frac{\partial}{\partial t} \int_{0}^{z} U_{\delta}(t,\xi) d\xi = f(t),$$
(53a)

where f(t) characterizes the intensity of incoming radiation (cf. Eq. (49)). The diffuse component, $U_d(t, z, \underline{\Omega})$, satisfies the following equations:

$$\begin{cases} U_{d}(t, z, \underline{\Omega}) + \frac{\sigma(\underline{\Omega})}{|\mu(\underline{\Omega})|} \int_{0}^{z} K(z, \xi, \underline{\Omega}) U_{d}(t, \xi, \underline{\Omega}) d\xi + \frac{1}{|\mu(\underline{\Omega})|} \frac{1}{c} \frac{\partial}{\partial t} \int_{0}^{z} U_{d}(t, \xi, \underline{\Omega}) d\xi \\ = \frac{1}{|\mu(\underline{\Omega})|} \int_{0}^{z} K(z, \xi, \underline{\Omega}) S(t, \xi, \underline{\Omega}) d\xi \\ + \frac{\sigma_{\underline{S}}(\underline{\Omega}_{0} \to \underline{\Omega})}{|\mu(\underline{\Omega})|} \int_{0}^{z} K(z, \xi, \underline{\Omega}) U_{\delta}(t, \xi) d\xi, \quad \mu(\underline{\Omega}) < 0, \end{cases}$$
(53b)
$$U_{d}(t, z, \underline{\Omega}) + \frac{\sigma(\underline{\Omega})}{\mu(\underline{\Omega})} \int_{z}^{H} K(z, \xi, \underline{\Omega}) U_{d}(t, \xi, \underline{\Omega}) d\xi + \frac{1}{\mu(\underline{\Omega})} \frac{1}{c} \frac{\partial}{\partial t} \int_{z}^{H} U_{d}(t, \xi, \underline{\Omega}) d\xi \\ = \frac{1}{\mu(\underline{\Omega})} \int_{z}^{H} K(z, \xi, \underline{\Omega}) S(t, \xi, \underline{\Omega}) d\xi + \frac{\sigma_{\underline{S}}(\underline{\Omega}_{0} \to \underline{\Omega})}{\mu(\underline{\Omega})} \int_{z}^{H} K(z, \xi, \underline{\Omega}) U_{\delta}(t, \xi) d\xi \\ + \bar{I}(t, H, \underline{\Omega}), \quad \mu(\underline{\Omega}) > 0, \end{cases}$$

where

$$S(t,\xi,\underline{\Omega}) = \int_{4\pi} \sigma_{S}(\underline{\Omega'} \to \underline{\Omega}) U_{d}(t,\xi,\underline{\Omega'}) d\underline{\Omega'}.$$
 (53c)

The mean intensity over the total space of the horizontal plane, $\bar{I}(t, z, \underline{\Omega})$, can be decomposed into direct, $\bar{I}_{\delta}(t, z)$, and diffuse, $\bar{I}_{d}(t, z, \underline{\Omega})$, components similarly:

$$\bar{I}_{\delta}(t,z) + \frac{\sigma(\underline{\Omega}_{0})}{|\mu(\underline{\Omega}_{0})|} \int_{0}^{z} p(\xi) U_{\delta}(t,\xi) d\xi + \frac{1}{|\mu(\underline{\Omega}_{0})|} \frac{1}{c} \frac{\partial}{\partial t} \int_{0}^{z} \bar{I}_{\delta}(t,\xi) d\xi = f(t).$$
(54a)

and,

$$\begin{split} &\left[\bar{I}_{d}(t,z,\underline{\Omega}) + \frac{\sigma(\underline{\Omega})}{|\mu(\underline{\Omega})|} \int_{0}^{z} p(\xi) U_{d}(t,\xi,\underline{\Omega}) d\xi + \frac{1}{|\mu(\underline{\Omega})|} \frac{1}{c} \frac{\partial}{\partial t} \int_{0}^{z} \bar{I}_{d}(t,\xi,\underline{\Omega}) d\xi \\ &= \frac{1}{|\mu(\underline{\Omega})|} \int_{0}^{z} p(\xi) S(t,\xi,\underline{\Omega}) d\xi + \frac{\sigma_{S}(\underline{\Omega}_{0} \to \underline{\Omega})}{|\mu(\underline{\Omega})|} \int_{0}^{z} p(\xi) U_{\delta}(t,\xi) d\xi, \quad \mu(\underline{\Omega}) < 0, \\ &\left\{\bar{I}_{d}(t,z,\underline{\Omega}) + \frac{\sigma(\underline{\Omega})}{\mu(\underline{\Omega})} \int_{z}^{H} p(\xi) U_{d}(t,\xi,\underline{\Omega}) d\xi + \frac{1}{\mu(\underline{\Omega})} \frac{1}{c} \frac{\partial}{\partial t} \int_{z}^{H} \bar{I}_{d}(t,\xi,\underline{\Omega}) d\xi \\ &= \frac{1}{\mu(\underline{\Omega})} \int_{z}^{H} p(\xi) S(t,\xi,\underline{\Omega}) d\xi + \frac{\sigma_{S}(\underline{\Omega}_{0} \to \underline{\Omega})}{\mu(\underline{\Omega})} \int_{z}^{H} p(\xi) U_{\delta}(t,\xi) d\xi \\ &+ I(t,H,\underline{\Omega}), \quad \mu(\underline{\Omega}) > 0. \end{split}$$

$$(54b)$$

where, $S^{(j)}(t,\xi,\underline{\Omega})$ is defined by Eq. (53c).

Numerical Scheme: The numerical scheme of solutions of the time-dependant stochastic RT is an extension of the corresponding scheme for the stationary stochastic RT equation for single species (Chapter 7). The key steps are the same. The mean intensity over whole horizontal plane, $\overline{I}(t, z, \Omega)$, is derived based on mean intensity over vegetation, $U(t, z, \Omega)$ (Eq. (50)). The scheme to solve for $U(t, z, \Omega)$ involves two steps: 1) solve Eq. (53a) for direct intensity, $U_{\delta}(t, z, \Omega_0)$; 2) solve Eq. (53b) for diffuse intensity, $U_d(t, z, \Omega)$. To solve for direct intensity, Eq. (53a) is integrated over time step, $[t_{i-1}, t_i]$, and the obtained integrals are approximated with a trapezoidal quadrature scheme. Next, terms that depend on t_{i-1} are grouped, (to be referenced below as $F_{\delta}(t_{i-1}, z, \Omega_0)$), and an integral equation for U_{δ} is formulate with the remaining terms at time t_i ,

$$U_{\delta}(t_{i},z) + \frac{\sigma(\underline{\Omega}_{0})}{\left|\mu(\underline{\Omega}_{0})\right|} \int_{0}^{z} K'(z,\xi,\underline{\Omega}_{0}) U_{\delta}(t_{i},\xi) d\xi = f(t_{i},\lambda) + F_{\delta}(t_{i-1},z,\underline{\Omega}_{0}), \qquad (55a)$$

where

$$K'(z,\xi,\underline{\Omega}_0) \equiv K(z,\xi,\underline{\Omega}_0) + (cW_t \sigma(\underline{\Omega}_0))^{-1},$$
(55b)

$$F_{\delta}(t_{i-1}, z, \underline{\Omega}_{0}) \equiv f(t_{i-1}, \lambda) - U_{\delta}(t_{i-1}, z) - \frac{\sigma(\underline{\Omega}_{0})}{\left|\mu(\underline{\Omega}_{0})\right|} \int_{0}^{z} K''(z, \xi, \underline{\Omega}_{0}) U_{\delta}(t_{i-1}, \xi) d\xi,$$
(55c)

$$K''(z,\xi,\underline{\Omega}_0) \equiv K(z,\xi,\underline{\Omega}_0) - (cW_t \sigma(\underline{\Omega}_0))^{-1}.$$
(55d)

Here $W_t = 0.5 \cdot \Delta t$ is the weight that depends on the numerical scheme used to approximate the integrals over the interval $[t_{i-1}, t_i]$. Note that at each time step t_i , the solution of Eq. (55a), U_{δ} , depends on U_{δ} at the previous time step, t_{i-1} . Intensity U_{δ} at t=0 is given by initial conditions (Eq. (49)). At each following time step, Eq. (55a) is solved numerically according to scheme of solution of Volterra integral equation, detailed in Chapter 7 (Eq. (42)).

Next, diffuse intensity, $U_d(t, z, \underline{\Omega})$, is evaluated. Implementing the above described time-integration procedure with respect to Eq. (54), we get

$$\begin{cases} U_{d}(t_{i}, z, \underline{\Omega}) + \frac{\sigma(\underline{\Omega})}{|\mu(\underline{\Omega})|}_{0}^{z} K'(z, \xi, \underline{\Omega}) U_{d}(t_{i}, \xi, \underline{\Omega}) d\xi = \frac{1}{|\mu(\underline{\Omega})|}_{0}^{z} K(z, \xi, \underline{\Omega}) S(t_{i}, \xi, \underline{\Omega}) d\xi \\ + \frac{\sigma_{S}(\underline{\Omega}_{0} \rightarrow \underline{\Omega})}{|\mu(\underline{\Omega})|}_{0}^{z} K(z, \xi, \underline{\Omega}) U_{\delta}(t_{i}, \xi) d\xi + F_{d}(t_{i-1}, z, \underline{\Omega}), \quad \mu(\underline{\Omega}) < 0, \\ U_{d}(t_{i}, z, \underline{\Omega}) + \frac{\sigma(\underline{\Omega})}{\mu(\underline{\Omega})}_{z}^{H} K'(z, \xi, \underline{\Omega}) U_{d}(t_{i}, \xi, \underline{\Omega}) d\xi = \frac{1}{\mu(\underline{\Omega})}_{z}^{H} K(z, \xi, \underline{\Omega}) S(t_{i}, \xi, \underline{\Omega}) d\xi \\ + \frac{\sigma_{S}(\underline{\Omega}_{0} \rightarrow \underline{\Omega})}{\mu(\underline{\Omega})}_{z}^{H} K(z, \xi, \underline{\Omega}) U_{\delta}(t_{i}, \xi) d\xi + F_{d}(t_{i-1}, z, \underline{\Omega}) + I_{d}(t_{i}, H, \underline{\Omega}), \quad \mu(\underline{\Omega}) > 0. \end{cases}$$
(56)

Here $F_d(t_{i-1}, z, \Omega)$ includes, similar to $F_{\delta}(t_{i-1}, z, \Omega_0)$, all the terms of integration of Eq. (53b) which depends on t_{i-1} . The solution U_d at time t_i depends on U_d at previous time step, t_{i-1} . At each time step the right hand side of Eq. (55a) is evaluated from know $U_{\delta}(t_i)$ at this time step and $U_d(t_{i-1})$ at previous time step. Equation (55a) is solved at each time step, t_i , with method of successive orders of scattering approximations (SOSA, Chapter 7, Eq. (40)).

The descretization of the equations for the mean intensity over a whole horizontal plane (Eq. (54), direct and diffuse components) are similar to those for mean intensity over vegetation (cf. Eq. (55)). Namely, the discreate equation for the direct component is

$$\bar{I}_{\delta}(t_{i},z) + \frac{1}{cW_{t}|\mu(\underline{\Omega}_{0})|} \int_{0}^{z} \bar{I}_{\delta}(t_{i},\xi) d\xi = -\frac{\sigma(\underline{\Omega}_{0})}{|\mu(\underline{\Omega}_{0})|} \int_{0}^{z} p(\xi) U_{\delta}(t_{i},\xi) d\xi + f(t_{i}) + G_{\delta}(t_{i-1},z,\underline{\Omega}_{0}).$$
(57)

The discreate equation for diffuse component is

$$\begin{split} & \left[\bar{I}_{d}(t_{i}, z, \underline{\Omega}) + \frac{1}{cW_{t} | \mu(\underline{\Omega})|}_{0}^{z} \bar{I}_{d}(t_{i}, \xi, \underline{\Omega}) d\xi = G_{d}(t_{i-1}, z, \underline{\Omega}) \\ & + \frac{1}{|\mu(\underline{\Omega})|}_{0}^{z} \begin{bmatrix} -\sigma(\underline{\Omega})U_{d}(t_{i}, \xi, \underline{\Omega}) + \sigma_{S}(\underline{\Omega}_{0} \rightarrow \underline{\Omega})U_{\delta}(t_{i}, \xi) \\ + S(t_{i}, \xi, \underline{\Omega}) \end{bmatrix} p(\xi) d\xi, \quad \mu(\underline{\Omega}) < 0, \\ & \bar{I}_{d}(t_{i}, z, \underline{\Omega}) + \frac{1}{cW_{t}\mu(\underline{\Omega})}_{z}^{H} \bar{I}_{d}(t_{i}, \xi, \underline{\Omega}) d\xi = G_{d}(t_{i-1}, z, \underline{\Omega}) \\ & + \frac{1}{\mu(\underline{\Omega})}_{z}^{H} \begin{bmatrix} -\sigma(\underline{\Omega})U_{d}(t_{i}, \xi, \underline{\Omega}) + \sigma_{S}(\underline{\Omega}_{0} \rightarrow \underline{\Omega})U_{\delta}(t_{i}, \xi) \\ + S(t_{i}, \xi, \underline{\Omega}) \end{bmatrix} p(\xi) d\xi, \quad \mu(\underline{\Omega}) > 0. \end{split}$$
(58)

Functions $G_{\delta}(t_{i-1}, z, \underline{\Omega}_0)$ and $G_{d}(t_{i-1}, z, \underline{\Omega})$ are derived similarly to $F_{\delta}(t_{i-1}, z, \underline{\Omega}_0)$ and $F_{d}(t_{i-1}, z, \underline{\Omega})$. As Eq. (57) and (58) are structurally similar to Eqs. (55) and (56), the scheme of solution are also similar. Note, that in order to solve for \overline{I}_{δ} and \overline{I}_{d} , one needs to know U_{δ} and U_{d} .

3.3. Model Evaluation with Field Measurements

Field Measurements: A SLICER instrument measurements over four coniferous forests and mixed-deciduous sites in North America were selected to evaluate performance of the TSRT model. Ground measurements of vegetation parameters are available from multiple field campaigns performed at those sites. The characteristics of forests stands at each site are summarized in Table 1 and brief description of measurements is given below.

Table 1. Characteristics of the four sites used for model parameterization. SOJP and SOBS are southern old jack pine and black spruce stands measured during the BOREAS field campaign in 1996. The SERC sites are mixed deciduous forest stands at the Smithsonian Environmental Research Center (SERC), with the overstory predominantly comprised of tulip poplar.

Site	Age, years	Density, stems/ha	Tree height H, m	Crown length L, m	Leaf refl.	Leaf transm.	LAI	Ground refl.	Clump. Index
SOJP	60-75	1600-4000	16.5-18.5	11.0-12.3	0.53	0.32	2.61	0.330	0.5
SOBS	0-155	3700-5800	10.0-11.0	6.7-7.3	0.41	0.32	4.00	0.303	0.5
SERC (interm)	41	840	31.5-33.0	21.0-22.0	0.45	0.45	5.16	0.225	0.7
SERC (mature)	99	1187	32.0-36.5	21.3-24.3	0.45	0.45	5.26	0.225	0.7

The first two sites, southern old jack pine (*Pinus banksiana*), (SOJP, 53⁰55' N, 104⁰42' W), and southern old black spruce (*Picea mariana*), (SOBS, 53⁰59' N, 105⁰07' W), are located in the BOREAS southern study area in Saskatchewan, Canada. These sites were the subjects of intensive field campaigns carried out in 1994 as part of BOREAS field activities. A field data on forest age, stem density, tree structural parameters (height, LAI, crown length, foliage area volume density), and optical properties of leaves and ground, were collected [Chen et al., 1997; Middleton et al., 1997; Ni-Meister, 2001]. SLICER measurements over the BOREAS southern area, including SOJP and SOBS, sites were taken on July 29, 1996. We neglect change in the stands biophysical parameters during the two-year period between the field and SLICER measurements.

The other two sites are closed-canopy stands located in a mixed deciduous forest at the Smithsonian Environmental Research Center (SERC, 38⁰53' N, 76⁰33' W), about 10 km south-

southeast of Annapolis, MD, on the western shore of Chesapeake Bay [Harding et al., 2001]. The stands belong to the tulip poplar association, a mixed-deciduous forest with the overstory predominantly comprised of tulip poplar (*Liriodendron tulipifera*). The other species include sweet gum, oak, hickory, and American beech. The stands represent distinct stages in canopy development. The intermediate stand is a broad unimodal canopy consisting of 14 different species. The mature stand has a bimodal vertical leaf area structure and consists of 19 species. The gap fraction is relatively small: 0.05 for the intermediate stand and 0.02 for the mature stand. SLICER data for these sites were collected on September 7, 1995. Ground measurements were taken within two weeks after that [Harding et al., 2001]. The trees were fully leaved at this time. Canopy structural parameters (forest age, stem density, tree heights and LAI) are listed in [Harding et al., 2001]; crown lengths are assumed to be two third of tree heights [Oliver & Larson, 1990]; leaf optical properties are taken from [Parker et al., 2001]. The ground surface is dominantly comprised of leaf-litter with some bare soil and rare live foliage. The reflectance of the ground is assumed to be half that of the canopy.

Selection of Model Parameters: The stochastic model parameters were selected based on values given in Table 1 for each site. Below two additional modeling issues are discussed: 1) representation of lidar signal; 2) representation of stochastic canopy structure. SLICER lidar pulses are characterized by a curved "Raleigh" shape and a width of 4 ns [Harding et al., 2000]. The vertical resolution of recorded waveforms is 0.112 m. We smoothed SLICER signals to obtain the resolution of 0.336 m. The amount of transmitted energy per pulse is 0.7 mJ. The canopy is divided into n layers of thickness, $\Delta z = 0.336$ m. The observation time is divided into m intervals, $[t_{i-1}, t_i]$, $\Delta t = t_i - t_{i-1} = \Delta z/c$ (c is the speed of light). The simulated Raleigh pulse is divided into k narrow pulses of duration, Δt . At each moment, t_i , i=[0;m], the intensity of incoming radiation is equal to the pulse amplitude at that moment.

The canopy structure was modeled as follows. The first moment of canopy structure, probability to find vegetation elements at height z, p(z), was implemented based on the assumption of hemi-ellipsoid crown shapes and field-measured values of LAI:

$$p(z) = p_0 \left[1 - \frac{(z - L)^2}{L^2} \right],$$
(59)

where $p_0 \equiv 3LAI/(2Lu_L)$, was derived according to definition (Chapter 7, Eq. (2)). Small variation of p-values (within 10%) was applied to each layer to provide a more realistic representation of canopy structure. The second moment of canopy structure, the conditional pair-correlation function, $K(z,\xi,\Omega)$, was implemented according to turbid medium approximations, adjusted for canopy clumping, namely,

$$K(z,\xi,\underline{\Omega}) = \beta \frac{p(z)p(\xi)}{p(z)} = \beta p(\xi).$$
(60)

Here β is the clumping index [Chen et al., 1997]: $\beta=1$ for uniformly distributed foliage (turbid medium); $\beta>1$ if leaves are regularly distributed (i.e leaves are all laid side by side); $\beta<1$ if leaves are clumped. Canopy clumping is ad-hook parameter, commonly used to adjust extinction coefficient [Chen et al., 1997]. In fact, Eq. (60) is equivalent to Chen et al. [1997] as K(z, ξ, Ω) is coupled with extinction coefficient in the stochastic equations (cf. Eq. (51)). Physical meaning of the Eq. (60) is as follows: clumping reduces the probability of finding vegetated elements in two layers simultaneously moving along a given direction.

Sensitivity Study: The following problems were analyzed: 1) the behavior of the model as function of the laser pulse duration; 2) the sensitivity of the model to the number of iterations. The model parameters used in this study correspond to BOREAS SOBS site (cf. Table 1). With respect to the first problem, note that, if a constant photon flux is incident on the top of vegetation over an extended period of time, the solution of the time-dependent stochastic RT equation converges to the solution of the stationary equation (Chapter 7). Figure 13 illustrates the convergence of the upward diffuse radiation flux at the top of canopy evaluated with the time-dependent model to that of stationary model. It requires about 60 ns to reach the stationary limit in this particular case. During this time, photons travel a distance of 20 m, which is approximately equal to 2H. Figure 13 also demonstrates that the use of a steady-state radiative transfer in optical remote sensing is valid. Namely, the time for radiation to propagate through the system (tens of ns) is short compared to variations in illumination conditions (typically, tens of minutes) or canopy geometry (seconds). The ability of the model to describe non-stationary and stationary radiation regimes makes it valuable for interpretation of both active and passive remote sensing data.

Figure 13. Upward diffuse radiation at the top of canopy as a function of time. The solid line represents the simulations by the stationarry model. The dotted line corresponds to simulations by the time-dependent model with increasing pulse width $(pw \rightarrow inf)$.

Second, the contribution of photons scattered n times to the radiation regime at a given moment of time is evaluated. The integral

$$E_{n}(t) = \int_{0}^{L} dz \int_{4\pi} d\underline{\Omega} J_{n}(t, z, \underline{\Omega})$$
(61)

is taken as a measure of the number of photons, scattered n times within the vegetation canopy at moment t. Here $J_n(t_i, z, \underline{\Omega})$ is the intensity of photons scattered n times. Figure 14 shows the ratios

$$-\ln \frac{\mathrm{E_n}(\mathrm{t})}{\mathrm{E_1}(\mathrm{t})}$$
 and $-\ln \frac{\mathrm{E_{n+1}}(\mathrm{t})}{\mathrm{E_n}(\mathrm{t})}$

as functions of the scattering order, n, for different instants of time, t. The former shows the proportion of multiply-scattered to single-scattered photons, while the latter is the rate at which multiply scattered photons contribute to the radiation field. One can see that the contribution of high-order scattering decreases rapidly for $t \le 17$ ns. For a sufficiently large n, all curves can be asymptotically replaced by the following curves:

$$-\ln\frac{E_{n}(t)}{E_{1}(t)} = -n\ln\rho(t) - \ln c(t), \qquad (62)$$

where $-\ln \rho(t)$ is the slope ratio, and $-\ln c(t)$ is a function, characterizing the shift of the plotted curves with time. Note that for the cases with large values of time, the ratio $-\ln[E_n(t)/E_1(t)]$ first decreases for small n. It describes the situation when $E_n(t) > E_1(t)$. In the stationary case, the maximum number of photons scattered n times would not change with time due to stationary input of radiation into the canopy. In the time-dependant case, photon flux enters the canopy during a small period of time equal to the laser pulse duration. With time, some fraction of photons scattered only once leaves the canopy and number of multiple-scattered photons exceeds the number of single-scattered photons inside the canopy.

Figure 14. Contribution of photons scattered n times at different time moments (t = 10, 17, 25, 33, 40 ns). The laser pulse is assumed to be incident at the top of canopy at time t = 0. Function E_i is a measure of n umber of photons scattered I times.

It follows from Eq. (62) that, for a sufficiently large n, E_n can be approximated by

$$E_n(t) = (\rho(t))^n c(t).$$
 (63)

Here $(\rho(t))^n$ can be interpreted as the probability for a photon to be scattered n times and c(t) as a function characterizing the dependence on time. The rate of convergence of the SOSA method is defined as

$$E - E_n = E_{n+1} + E_{n+2} + \dots = c(t)(\rho(t))^{n+1}(1 + \rho(t) + \dots) = c(t)\frac{(\rho(t))^{n+1}}{1 - \rho(t)}.$$
(64)

Thus the contribution of multiply scattered photons to the radiation field at a given instant of time is proportional to $(\rho(t))^n / (1 - \rho(t))$. The higher the probability of scattering events, the higher the contribution of multiply scattered photons to the radiation field. In this example, about n = 10 iterations are required to obtain a relative accuracy of 10^{-3} .

Simulations of Lidar Waveforms: For each of the sites, six SLICER waveforms were extracted. The selection was made among data from the three interior footprints; waveforms corresponding to the two outer footprint positions were discarded as anomalous due to misalignment between the scan pattern and the outer edges of the instruments receiver field-of-view [Lefsky et al., 1999a].

The selected signals were normalized by the maximum return signal in each waveform. It is the ground return for SOBS and SOJP and the maximum canopy return for the SERC sites. The digitizer bin units were converted into distance, with one digitizer bin corresponding to 0.1112 m. A smoothing procedure was applied to the signals: three adjacent bins were summed and the averaged value was taken. The ground return was identified as the last discrete return above the mean background noise level. The maximum value of the ground return was assigned a height of zero. Tree height is the distance between the maximum ground return and the first canopy return. As the plotted SLICER signals reveal different heights for the same forest stand, the tree heights and crown sizes used in model simulations were defined for each signal individually.

Figures 15 and 16 summarize the results for SOBS/SOJP and SERC sites respectively. For convenience, each waveform is accompanied by the site name and a number. The gap fraction is high in the coniferous stands (SOJP and SOBS) and the return from the ground is pronounced most. The unimodal shapes of the SLICER waveforms over those sites imply the absence of secondary forest understory. The third SOBS plot reveals some understory, seen as a small wide spike near the maximum return. For the first SOBS plot, the amplitude of the SLICER signal is greater than that in the simulations. Differences in foliage density affect the strength of the SLICER signals, while the model simulations assume a constant value. The agreement between

the SLICER signals and model simulations is better for the signals with a slow decay of the waveform. The SOBS SLICER signals shown in Fig. 15 were collected over the region where ground measurements described in Chen et al. [1997] were made. For the SOJP site, none of the SLICER tracks coincides with the location of Chen et al.'s measurements. Two SLICER tracks cross the area of another field campaign described in [Sun & Ranson, 2000]. The data from these tracks are used here. For the SERC forest stands, the SLICER data sets include regions where ground measurements were taken. Several subpeaks of canopy return, and small or even undistinguishable ground returns are typical for the waveforms collected over both mature and intermediate stands. The signals with identifiable ground returns were chosen to calculate tree heights. Only relatively unimodal waveform profiles are represented in Fig. 16, for the model does not include multi-layer vegetation structure. Some small understory, in the second and third mature and intermediate plots, is acceptable. The majority of radiation is intercepted in the upper layer of the canopy in view of high foliage density in these stands.

Figure 15. Comparison of model simulations (dotted curves) and corresponding SLICER measurements (solid curves) for BOREAS SOBS and SOJP sites. The noise level is shown as a vertical dashed line.

Figure 16. Comparison of model simulations (dotted curves) and corresponding SLICER measurements (solid curves) for SERC mature and intermediate forest stands. The noise level is shown as a vertical dashed line.

3.4. Effect of Multiple Scattering

The simulated returns generated from single and multiple scattering are next compared to evaluate the contribution of multiply scattered photons. From the plots shown in Fig. 15 and 16, those that agreed best with SLICER data were chosen for this investigation; one plot for each forest stand. These are SOBS (2), SOJP (1), Mature (3), and Intermediate (1). The model-simulated signals in the single scattering approximation were added to each plot. The results are shown in Fig. 17.

The inclusion of multiple scattering magnifies the signal and enhances significantly the lower part of the waveform. In general, multiply scattered photons carry information on canopy structural parameters, namely, foliage density and gap fraction. When a photon path in the canopy is limited to a single interaction, the total travel time is the round trip time between the sensor and the interaction point: $t_{total} = t_{ps}$. If a photon has interacted several times before leaving the canopy in the direction of the sensor, the total travel time is increased to $t_{total} = t_{ps} + t_{pm}$, where t_{pm} is the extra time due to multiple scattering. This extra time depends on two variables: the number of interactions and the mean distance of photon travel between two interactions (photon mean free path). The photon mean free path is inversely proportional to the extinction coefficient $\sigma(\underline{r}, \underline{\Omega})$, which is directly related to canopy structure. A vegetation canopy with high density will generate more multiple scattering but with relatively short free paths.

Figure 17. SLICER waveforms and model simulations with and without multiple scattering for four different sites. The solid curves represent the SLICER signals. The single scattering simulations are shown with dashed curves, while the dotted represent multiple curves scattering simulations. Noise is subtracted.

In case of sparse canopies, single-scattering approximation models are expected to provide good approximation of lidar recorded signals. A hybrid geometric optical and radiative transfer model (GORT) was tested with data from four BOREAS sites: SOBS, SOJP, NOBS (northern old black spruce), and NOJP (northern old jack pine) [Ni-Meister et al., 2001]. Selected SLICER

waveforms were normalized by the maximum signal (ground return). The model provided good agreement between the measured and simulated waveforms, but the relative amplitude of vegetation return was low: less than 0.15 for the SOBS site and less than 0.1 for the others. This indicates that the density of measured vegetation was relatively low and the effects of multiple scattering were probably insignificant. For the study presented here, the lower boundary of relative amplitude was 0.3 for SOJP and 0.4 for SOBS.

The waveform shapes change dramatically in a set of measurements for the same site. It depends on the density of vegetation and the number of trees within the footprint. As a result, some waveforms may show effects of multiple scattering, while the others may not. The effect of multiple scattering also depends on the amplitude and duration of a laser pulse. The model shows that this effect is more significant in the simulations with longer pulse widths. An increase of the amplitude makes it more detectable.

3.5. Summary

The application of the stochastic approach for the large footprint lidar remote sensing of heterogeneous vegetation was introduced. The Time-dependant Stochastic RT (TSRT) model describes non-stationary radiation regime of lidar measurements (the stream of photons, emitted by lidar, enters vegetation canopy, propagate through vegetation bounded by soil via mechanism of single and multiple scattering and exit canopy back to lidar receiver). Sensitivity study indicates that as the duration of lidar pulse exceed the time required for photon to travel the distance twice of canopy height, the solution of the TSRT model converges to one of the stationary stochastic equation. Also, contribution of the multiple-scattered photon with respect to single-scattered photons was quantified (how it changes with time and how it depends on the order of scattering). The simulated lidar waveforms were evaluated with respect to SLICER lidar measurements and ancillary ground measurements of vegetation biophysical parameters over four coniferous and mixed-deciduous stands. The inclusion in the TSRT simulations of multiple scattering magnifies the signal and enhances significantly the lower part of the waveform. In general, multiply- scattered photons carry information on canopy structural parameters, namely, foliage density and gap fraction. A vegetation canopy with high density will generate more multiple scattering but with relatively short free paths. Conversely, in sparse canopies, the contribution of multiple scattering will be relatively lower but with longer paths. In case of sparse canopies, single-scattering approximation models are expected to provide good approximation of lidar recorded signals. The effect of multiple scattering also depends on the amplitude and duration of a laser pulse. This effect is more significant in the simulations with longer pulse widths. Further modeling efforts are required to improve representation of the conditional pair-correlation function (inclusion of stem density, canopy roughness, canopy layering, leaf distribution) in order to better capture relationship between canopy structure and lidar waveforms.

References

- Blair, J.B., and M.A. Hofton (1999). Modeling laser altimeter return waveforms over complex vegetation using high-resolution elevation data. *Geophys. Res. Lett.*, **26**, 2509-2512.
- Borel, C., and S. Gerstl (1994). Nonlinear spectral mixing models for vegetative and soil surfaces. *Remote Sens. of Environ.*, **47**, 403-416.
- Breda, N.J.J. (2003). Ground-based measurements of Leaf Area Index: a review of methods, instruments and current controversies. J. Exp. Botany, 64(392), 2403-2417.
- Carpenter, G.A., S. Gopal, S. Macomber, S. Martens, and C.E.Woodcock (1999). A neural network method for mixture estimation for vegetation mapping. *Remote Sens. Environ.*, 70, 135-152.
- Chen, J.M., P.M. Rich, S.T. Gower, J.M. Norman, and S. Plummer (1997). Leaf area index of boreal forests: Theory, techniques, and measurements. *J. Geophys. Res.*, **102** (**D24**), 29429-29443.
- DeFries, R.S., J.R.G Townshend, M.C. Hansen (1999). Continuous fields of vegetation characteristics at the global scale 1-km resolution. J. Geophys. Res.-Atmosphere, **104(D14)**, 16911-16923.
- Drake, J.B., R. Dubayah, D.B. Clark, R.G. Knox, J.B. Blair, M.A. Hofton, R.L. Chazdon, J.F. Weishampel, and S. Prince (2002a). Estimation of tropical forest structural characteristics using large-footprint lidar. *Remote Sens. Environ.*, **79**, 305-319.
- Drake, J.B., R. Dubayah, R.G. Knox, D.B. Clark, and J.B. Blair (2002b). Sensitivity of large-footprint lidar to canopy structure and biomass in a neotropical forest. *Remote Sens. Environ.*, **81**, 378-392.
- Dubayah, R., R. Knox, M. Hofton, J.B. Blair, and J. Drake (2000). Land surface characterization using lidar remote sensing, in Spatial Information for Land Use Management, Hill, M., and Aspinall, R. (Eds.), Gordon and Breach, Newark, NJ.
- Ju J., E.D. Kolaczyk, and S. Gopal (2003). Gaussian mixture discriminant analysis and sub-pixel land cover characterization in remote sensing. Remote Sens. Environ. **84**, 550-560.
- Govaerts, Y. M. (1996). A model of light scattering in three-dimensional plant canopies: a Monte Carlo ray tracing approach, Space Applications Institute, JRC, European Commission.
- Harding, D.J., J.B. Blair, D.L. Rabine, and K.L. Still (2000). SLICER airborne laser altimeter characterization of canopy structure and sub-canopy topography for the BOREAS northern and southern study regions: Instrument and data product description, in Technical report series on the Boreal Ecosystem-Atmospheric Study (BOREAS), Hall F.G., and Nickeson, J. (Eds.) Washington, DC: NASA/TM-2000-209891.
- Harding, D.J., M.A. Lefsky, G.G. Parker, and J.B. Blair (2001). Laser altimeter canopy height profiles. Methods and validation for closed-canopy, broadleaf forests. *Remote Sens. Environ.*, **76**, 283-297.
- Huang D., Y. Knyazikhin, W. Yang, W. Wang, J. Privette, D. Deering, N. Shabanov, and R.B. Myneni (2007). Stochastic transport theory for investigating the three-dimensional canopy structure from space measurements. *Remote Sens. Environ. in press.*
- Kotchenova, S.Y., N.V. Shabanov, Y. Knyazikhin, A. Davis, R. Dubayah, and R.B. Myneni (2003). Modeling lidar waveforms with time-dependent stochastic radiative transfer theory for remote estimates of forest structure. J. Geophys. Res.- Atmospheres, 108(D15), 4484, doi:10.1029/2002JD003288.

- Lefsky, M.A., D.J. Harding, W.B. Cohen, G.G. Parker, and H.H. Shugart (1999a). Surface lidar remote sensing of basal area and biomass in deciduous forests of eastern Maryland, USA, *Remote Sens. Environ.*, **67**, 83-98.
- Lefsky, M.A., W.B. Cohen, S.A. Acker, G.G. Parker, T.A. Spies, and D. Harding (1999b). Lidar remote sensing of the canopy structure and biophysical properties of Douglas-fir western hemlock forests. *Remote Sens. Environ.*, **70**, 339-361.
- Lefsky, M.A., W.B. Cohen, G.G. Parker, and D.J. Harding (2002). Lidar remote sensing for ecosystem studies. *Bioscience*, **52**(1), 19-30.
- MacArthur, R.H., and H.C. Horn (1969). Foliage profiles by vertical measurements. *Ecology*, **50**, 802-804.
- McIver D.K, and M.A. Friedl (2002). Using prior probabilities in decision-tree classification of remotely sensed data. Remote Sens. Environ., 81, 253-261.
- Middleton, E.M., S.S. Chan, R.J. Rusin, and S.K. Mitchell (1997). Optical properties of black spruce and jack pine needles at BOREAS sites in Saskatchewan, Canada. *Canadian J. Remote Sens.*, **23**(2), 108-119.
- Ni-Meister, W., D.L.B. Jupp, and R. Dubayah (2001). Modeling lidar waveforms in heterogeneous and discrete canopies, *IEEE Trans. Geosci. Remote Sens.*, **39**(**9**), 1943-1958.
- Oliver, C.D., and B.C. Larson (1990). Forest stand dynamics, McGraw-Hill, Inc., New-York.
- Parker, G.G., M.A. Lefsky, and D.J. Harding, (2001). Light transmittance in forest canopies determined using airborne laser altimetry and in-canopy quantum measurements", *Remote Sens. Environ.*, **73**, 298-310.
- Pomraning, G.C. (1991). Linear kinetic theory and particle transport in stochastic mixtures. Series on advances in mathematics for applied sciences.-Vol. 7. Singapore: World Scientific.
- Press, W.H., B.P. Flannery, S.A. Teukolsky, and W.T. Vetterling. (1986). *Numerical recipes: the art of scientific computing*. p.19-38, Cambridge University Press.
- Ross ,J. (1981). *The radiation regime and architecture of plant stands*. Hague: Dr. W. Junk Publishers, 1981.
- Shabanov, N.V., Y. Knyazikhin, F. Baret, and R.B. Myneni (2000). Stochastic modeling of radiation regime in discontinuous vegetation canopies. *Remote Sens. Environ.*, 74(1), 125-144.
- Shabanov, N.V., D. Huang, W. Yang, B. Tan, Y. Knyazikhin, R.B. Myneni, D.E. Ahl, S.T. Gower, A. Huete, L.E. Aragao, and Y.E. Shimabukuro (2005). Analysis and optimization of the MODIS LAI and algorithm retrievals over broadleaf forests. *IEEE Trans. Geosci. Remote Sens.*, 43(8), 1855-1865.
- Shabanov, N.V., D. Huang, R.E. Dickinson, Y. Knyazikhin, and R.B. Myneni (2007). Stochastic radiative transfer model for mixture of discontinuous vegetation canopies. J. Quant. Spectroscopy and Radiative Transf.; in press.
- Stoyan, D., W.S. Kendall, and J. Mecke (1995). *Stochastic geometry and its applications*. p.67-69, Wiley Series in Probability and Statistics, Second Ed., John Wiley & Sons.
- Sun, G., and K.J. Ranson (2000). Modeling lidar returns from forest canopies, *IEEE Trans. Geosci. Remote Sens.*, **38(6)**, 2617-2626.
- Tian Y, Y. Wang, Y. Zhang, Y. Knyazikhin, J. Bogaert, and R.B. Myneni (2002). Radiative transfer based scaling of LAI/FPAR retrievals from reflectance data of different resolutions. *Remote Sens. Environ.*; **84**, 143-159.

- Tian. Y, R.E. Dickinson, L. Zhou, X. Zeng, Y. Dai, R.B. Myneni, Y. Knyazikhin, X. Zhang, M. Friedl, H. Yu, W. Wu, and M. Shaikh (2004). Comparison of seasonal and spatial variations of LAI/FPAR from MODIS and Common Land Model. J. Geophys. Res.-Atmosphere, 109(D1), D01103, 10.1029/2003JD003777.
- Zwally, H.J., B. Schutz, W. Abdalati, J. Abshire, C. Bentley, A. Brenner, J. Bufton, J. Dezio, D. Hancock, D. Harding, T. Herring, B. Minster, K. Quinn, S. Palm, J. Spinhirne, and R. Thomas (2002). ICESat's laser measurements of polar ice, atmosphere, ocean and land. J. *Geodynamics*, 34, 405-445.